The Values Distribution in a Competing Shares Financial Market Model

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Abstract. We present our competing shares financial market model and describe it's behaviour by numerical simulation. We show that in the critical region the distribution avalanches of the market value as defined in this model has a power-law distribution with exponent around 2.3. In this region the price returns distribution is truncated Levy stable.

INTRODUCTION AND MODEL

Recent studies [5] of the S&P500 financial market index have shown that financial markets are not appropriately described by the Efficient Market Hypothesis which predicts a Gaussian type behaviour of the price changes time series. [11] In this paper we will present a reinterpretation of our toy financial market model which was presented in [2]. That model was a trader based model, here however we re-present that model as a share based system. That model was a simplified mean-field version of our neural-network type financial market model presented in [1]. Although this model was developed independently, it is a variant of the Minority Game [6], with however some fundamental differences. We explain this model here as follows.

There are N shares labeled $i = 1,..,N$ (or commodities etc) in a competetive market. Each share at time t can be in one of two states $s_i(t)$ which classify the majority ownership crowd of the share i, $s_i(t) = 1$ 'bull', $s_i(t) = -1$ 'bear'. Bulls have bought the share, hoping to sell it later at a profit, bears have sold the share hoping to buy it back later at a profit. The price returns of each share are given by,

$$\Delta p_i(t) = \frac{N}{2} s_i(t). \tag{1}$$

that is like excess demand, with price returns change being proportional to the 'size' of the market N. Each share also has a *generally perceived value* $V_i(t)$, which ranks the shares i. High valued shares have recognized long term price trend with therefore *stuck*, low volatility, states $s_i(t)$. Low valued shares on the other hand are short term *risky* states with *high volatility*. The state update dynamic is Darwinian competition; we choose two shares at random, say a and b with $V_a(t) > V_b(t)$, then,

$$s_i(t+1) = \begin{cases} s_i(t), & (a) \\ -s_i(t), & (b) \end{cases} \tag{2}$$

The shares therefore define a jammed flow of investors, long term investors up the values, short term investors down the values. The high valued shares become stuck in bull(bear) states because nobody wants to sell(buy) them. In other words investors looking for long term investments look for the low volatility high valued shares, and therefore reinforce these shares price trends by trying to take the corresponding position on them. Low valued shares on the other hand are left in fluctuating states without any well-defined price trend and therefore have high-volatilty since nobody knows which way they are going. In this model therefore, both price trend and high volatilty self-reinforce themselves. Hence the spin update dynamic Eq.2, which basically says that high valued are low volatility and low valued high volatility.

In reality a shares value is a complex function of individual company news (affecting each share separately), macroscropic news(affecting all shares), personal traits, the weather etc. However in this model we model only the speculative behaviour of traders. The shares values are therefore defined as follows,

$$V_i(t+1) = V_i(t) - \frac{1}{2}\Delta s_i(t) G(t) - (1 - \frac{1}{2}|\Delta s_i(t)|)c \tag{3}$$

where $\Delta s_i(t) = s_i(t+1) - s_i(t)$ and our variable 'groupthink' $G(t)$ is the overall market (macroeconomic) state,$G(t) = \frac{1}{N}\sum_1^N s_i(t) = \frac{2\Delta p(t)}{N}$ and $p(t)$ is the share index price return. Therefore the stuck shares (a) have slowly decreasing value ($c > 0$) due to rise(fall) in the price of bull(bear) states. Risky shares (b) however, which are the domain of speculators and gamblers, increase in value when they move into the minority state of the *overall* market. Here we are assuming that the macroeconomic bull/bear state $G(t)$ is coupled to the individual share states $s_i(t)$ in the same way as the usual Minority Game, except on a larger scale.

This values update rule Eq.3 implies that the two main observables for investors picking between shares are the individual shares volatilities, which are related to the first term, and the individual shares price trends, which are related to the second term. Indeed long term investors may 'play' the observed price trends, while short term speculators may 'play' the volatility.

Now we generalise the Darwinian evolution dynamic by defining a probability for dynamic a or b update in Eq.2,i.e.

$$Probability(a) = 1 - Probability(b) = \frac{1}{1 + e^{-2\beta v_i(t)}} \tag{4}$$

where β is a kind of inverse temperature parameter, and $v_i(t) = V_i(t) - V(t)$ is the *relative* value, with $V(t) = \frac{1}{N}\sum_1^N V_i(t)$ the mean-value (market-value). This implies that indeed high valued shares are low volatility and low valued shares high volatility, with a volatility gradient which depends on the parameter β. This simply replaces our dynamic of randomly choosing two shares (a) and (b) and comparing the values. This evolution dynamic is similar to co-evolution on coupled fitness landscapes [7–9], since the relative values $v_i(t)$ can change due to changes in $V_i(t)$ or in a co-evolutionary sense by changes in $V(t)$.

Our dynamic is then that we first calculate $G(t)$ and $V(t)$ and then we update

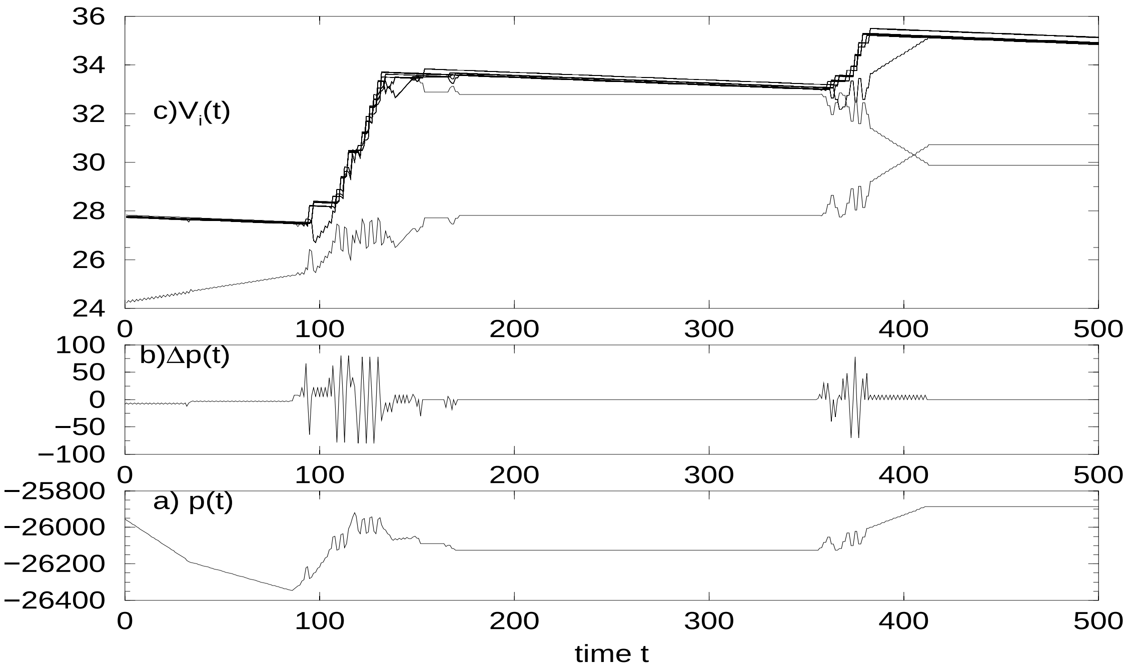

FIGURE 1. $c = 0.003$, $N = 80$ $\beta = \infty$ time series. a)Index price returns $p(t)$, b)Index price returns changes $\Delta p(t)$, c)Values $V_i(t)$

all shares $V_i(t)$ and $s_i(t)$ according to probability given by Eq.4. By putting $\beta = \infty$ we obtain the deterministic system where if $V_i(t) > V(t)$ then (a) and otherwise (b). For the initial conditions for which we choose $V_i(0) \in [-1, 1]$ uniformly randomly and $s_i(0) = \pm 1$ uniformly randomly.

A more general version of this model defines the price changes by extending the definition Eq.1, to,

$$\Delta p_i(t) = \frac{N}{2} s_i(t) v_i(t) \tag{5}$$

where $v_i(t)$ are the relative values, which we believe is more realistic than the simple definition Eq.1. However in this paper we confine to Eq.1, since the study of Eq.5 is still in progress. This model resembles the MG. However it is different in two fundamental ways. 1) We only apply the minority rule when the state changes, that is the amount a shares value is updated according to the minority rule is proportional to it's volatility across any time period. 2) *There is no strategy space.* We simply map straight from the values to the state update rule.

RESULTS

First we describe time series behaviour of the deterministic system [2,4]. Shown in Fig.1 are time series for $\beta = \infty$. For small c and large N the values $V_i(t)$ shows 'Punctuated Equilibrium' [10] type behaviour reminiscent of ecodynamics, i.e. avalanches and stasis periods.

In fact the avalanches are, to varying degrees, bursts of partial global synchronization where most of the share values $V_i(t)$ cluster into 2 or more groups and interleave the mean-value, growing rapidly in a co-operative way (see Fig.2), and represent an unstable attractor for the system. A few shares are left out and because this synchronization behaviour rapidly increases the values deviation $d(t) = \frac{1}{N}\sum_1^N v_i(t)$ we call these bursts 'flights to quality', like the phenomena which occur in real financial markets from time to time, such as the Russian Crisis in 1998.

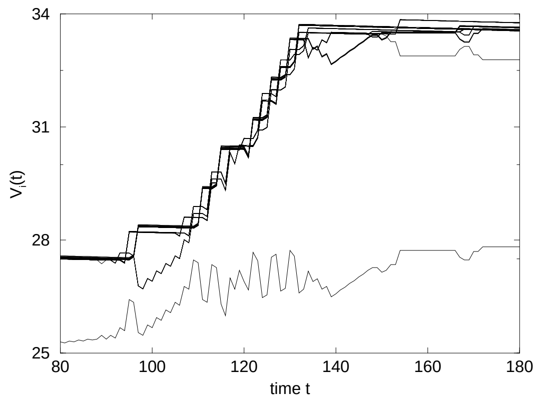

FIGURE 2. Detail of Values Time series from Fig.1

They occur due to the driving term c decreasing $d(t)$ and eventually causing one value to cross the mean-value thereby possibly starting an avalanche, and occur at small $d(t)$ when the market is susceptible to fluctuation (here internally generated by chaos). Indeed at times of small $d(t)$ nobody knows what is best to invest in and panics and stampedes may occur due to wild speculation (irrespective of the actual information contained in the item of news which hit the market). These bursts show up as oscillations (Fig.2(b)) in the price changes time series Δp, we call them 'market rollercoasters'. In fact the slightly oscillatory nature of financial time series has been noted [11,12]. The price time series shown (Fig.2(c)) is very reminiscent of

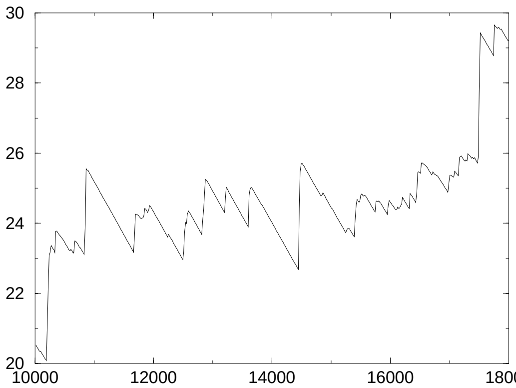

FIGURE 3. $N = 80$, $c = 0.003$, $\beta = \infty$, Mean-value $V(t)$ time series, (longer time length)

the technical-analysts 'double tops' and other recognized formations. In this view a market will interpret rumour as good or bad dependent on the current state $G(t)$, rather than on the value of the news itself, such as when the meeting of James Baker and Tariq Aziz before the Gulf War went on a little too long and caused a market rollercoaster. In fact when a market is over-bought (most people in bull states) the only way it can go is into sell mode and it is not so surprising that momentum may carry it into an over-sold state.

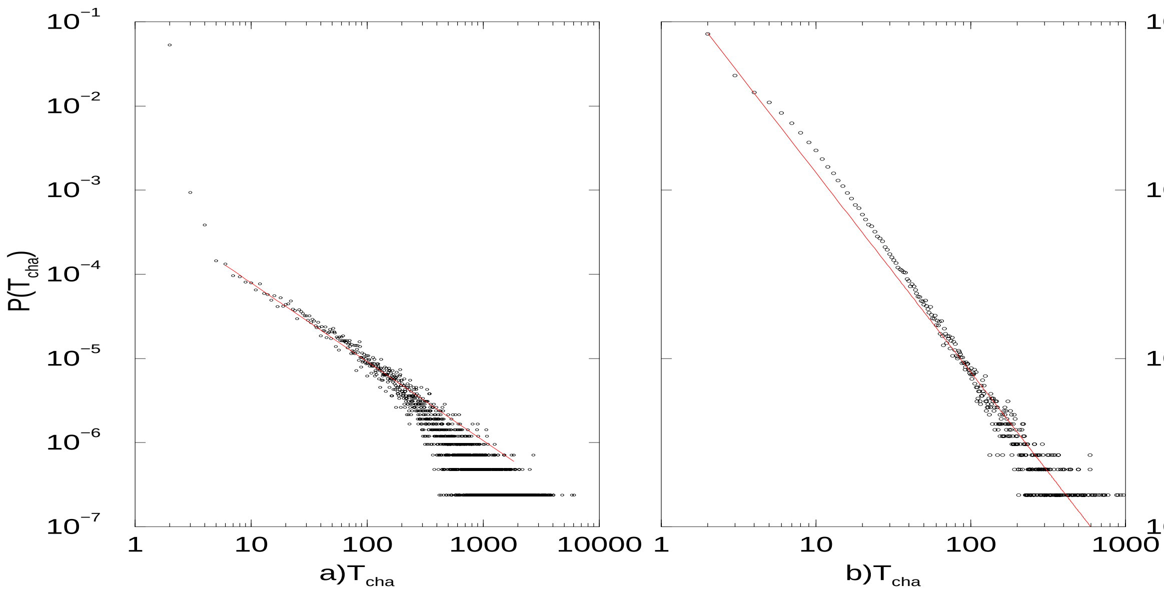

FIGURE 4. Probability distribution of lengths of periods of stasis, T_{cha}. a)Slope -0.93 ± 0.01. $c = 0.001$, $N = 200$, $\beta = \infty$, b)Slope -2.36 ± 0.03,$c = 0.001$, $N = 200$, $\beta = 80$.

The most fundamental variables in our model are the values rather than the prices and so we study their behaviour. A mean-values time series is shown in Fig.3. In [2] we showed that for small c this system is critical in the sense that avalanches, defined as size of changes in $V(t)$, have a power law distribution with a peak due to the almost periodic state. Here we show that the stasis periods also show such behaviour. Shown in Fig.4(a) is the distribution of T_{cha} where T_{cha} is the time between successive events $\Delta V(t) > 0$, they are normalized by dividing by the whole time series length $T_{len} = 4 \times 10^6$, after discarding a long transient. Only one time series is included in this figure, so periods of stasis of all sizes (limited by the system size) are always present. The slope of the line is very near 1, which is a good fit for the longer T_{cha} however at small T_{cha} there is an increase in probability due to time spent in the almost periodic states. This shows that total market value itself has a Punctuated Equilibrium time behaviour, independent of 'external' information. External information may however initiate an avalanche itself as mentioned above.

In fact more realistically we may study the stochastic system defined by setting $\beta \neq \infty$. We found that as we change β we see a phase transition, similar to that seen in the more general MG [6], of which this is model is a restricted version [3]. Furthermore in the transition region the price returns distribution defined by Eq.1 shows a Levy distribution [13,14] for the central values, while for values after about four standard deviations from the mean there is a drop-off of the probability. [3] Time series in this region at $\beta = 80$ are shown in Fig.5. The parameter characterising the Levy distribution we found was about 1.5 which is very similar to the actual distribution for the $S\&P500$ measured by Mantegna and Stanley [5]. Here we show that the corresponding mean-value changes ΔV distributions at $\beta = 80$ still show a good scaling behaviour, where however the exponents have changed. Shown in Fig.4(b) is the $\beta = 80$ T_{cha} distribution and in Fig.6 the distribution of changes ΔV, divided into both positive and negative contributions. Again only one time series of length $T_{len} = 4 \times 10^6$ was included. The exponents have changed from near 1 for the deterministic system to around -2.3 as shown in the figure captions. Recent work [15] has studied price returns distributions for the

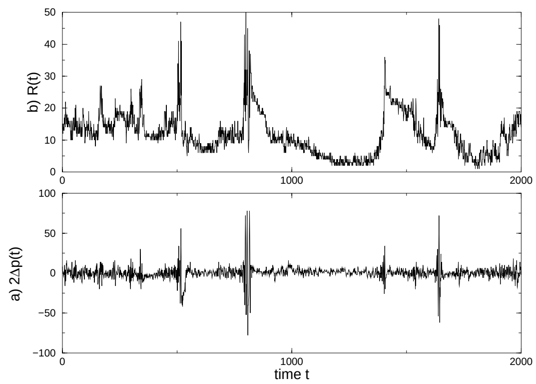

FIGURE 5. $N = 200$, $\beta = 80$, $c = 0.001$ a)Price changes time series, b)Trading volume $R(t)$ defined by the amount of spins which flip.

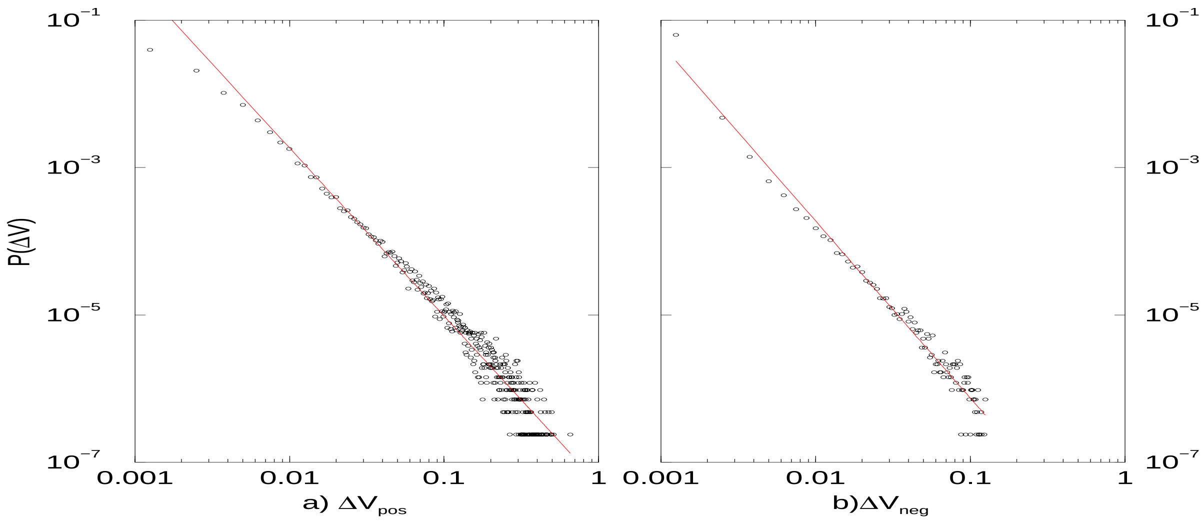

FIGURE 6. Probability distribution of avalanche sizes ΔV, $c = 0.001$, $N = 200$, $\beta = 80$, a)Positive changes, Slope -2.28 ± 0.03. $c = 0.001$, $N = 200$, $\beta = 80$, b)Negative changes slope -2.40 ± 0.05

$S\&P500$ and found that while the central region of the distribution may be characterised by a Levy stable distribution with parameter between 1.35-1.8 the tails fall off with an exponent of around 3. We are at present studying the behaviour of the tails of the price returns distributions for both Eq.1 and Eq.5, which may be more appropriate, to see if they give the correct results.

DISCUSSION

This model may seem naive but it seems to well-reproduce many observed characteristics of financial market time series, including qualitative features. Furthermore it is built on fairly simple assumptions and can explain results that other models based on a single share cannot, for example the fact that usually all shares crash together, and that all individual shares have similar distributions. [16]. This should be investigated further. This model predicts many interesting relationships between share prices, especially the values deviation $d(t)$. To what extent the behaviour of $d(t)$ corresponds with reality is under investigation.

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