The Complex Dynamics of a Simple Stock Market Model

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Abstract

We formulate a microscopic model of the stock market and study the resulting macroscopic phenomena via simulation. In a market of homogeneous investors periodic booms and crashes in stock price are obtained. When there are two types of investors in the market, differing only in their memory spans, we observe sharp irregular transitions between eras where one population dominates the market to eras where the other population dominates. When the number of investor subgroups is three the market undergoes a dramatic qualitative change - it becomes complex. We show that complexity is an intrinsic property of the stock market. This suggests an alternative to the widely accepted but empirically questionable random walk hypothesis.

Introduction

Most economic and financial models of the stock market discuss market equilibrium. According to the efficient market theory equilibrium stock prices reflect all the available information and all investors' preferences [1-6]. The notion of static equilibrium is in sharp contrast to the dynamic nature of the real market. The changes in stock prices are usually explained by asserting that new information is constantly supplied to the market and hence the equilibrium point is continuously shifting. According to the efficient market theory, since all the known information is already reflected in the current stock price, only new information moves stock prices. Then, the argument goes that because the new information is random we observe (or should observe, according to efficient market theory) prices following a random walk. Empirical evidence, however, has been accumulating against the random walk hypothesis [7-12].

In this paper we suggest a different explanation for the dynamic nature of the stock market. We show that in a market where investors use ex-post returns in order to estimate future returns on the stock, a static state may never be approached, even though no new external information is introduced.

We study the long range dynamics of a market of homogeneous investors, a market with two investor populations differing only in their memory span (the time span they look back at past returns), and a market with three investor populations. We show that it suffices to have three investor populations, differing only in their memory spans, in order to generate realistically complex price behaviour, even without any external influences (new information). The complexity is shown to be an intrinsic property of the market.

In section 1 we present the framework of our microscopic stock market model. As has been shown in [13-15] the homogeneous investor market, which is analougeous to a mean-field approximation, leads to an unrealistic macroscopic behaviour consisting of periodic booms and crashes in the stock price. The length of the boom-crash cycle is determined by the homogeneous memory span. This result is presented in section 2.

When there are two investor populations, with different memory spans, we observe the following different scenarios :

a) One population gains control over most of the money in the market and exclusively dictates the length of the boom-crash cycle.

b) One population becomes dominant, dictating market behaviour. After some time the second population abruptly takes over the market, and an era of dominance of the second population begins. This era continues untill the first population takes over again, and so on. Hence, we observe alternating distinct eras of dominance.

c) One population gains control over most of the money in the market, however, the market behaviour is surprisingly dictated by the other very poor population.

The ratio between the memory spans of the two investor types determines which of these scenarios actually takes place. We present and explain these results in section 3.

When there are three investor populations, one might expect to find rotation in market dominance between the three groups, as a natural extension of the two-population dynamics. This is generally not the case. When there are three investor subgroups the time series can no longer be devided into distinct eras where one of the populations dominates. Instead, all populations are acting simultaneously, with strong nonlinear coupling between them. The market behaviour becomes very complex, and more realistic. In section 4 we discuss the dynamics of a market with three and more investor subgroups.

We present our conclusions in section 5.

1. The Model

The microscopic 'element' of our model is the individual investor. Individual investors interact via the buying and selling of stocks and bonds. The model presented here is the most basic model attainable in which all the crucial elements of the stock market are included. We have consciously made certain simplifying assumptions and omitted some of the features of the real market. We explain our notations as we go along, but also give an organized list of notations for reference at the end of section 1.1.

Our stock market consists of two investment options: a stock (or index of stocks) and a bond. The bond is assumed to be a riskless asset, and the stock is a risky asset. The stock serves as a proxy for the market portfolio, (e.g., the Standard & Poors index). The extension from one risky asset to many risky assets is straightforward. However, one stock (the index) is sufficient for our present analysis because we restrict ourselves to global market phenomena and do not wish to deal with distributions across several risky assets. The investors are allowed to revise their portfolio at given time points, i.e. we discuss a discrete time model. The bond is assumed to be a riskless investment yielding a constant return at the end of each time period. The bond is exogenous and investors can buy from it as much as they wish at a given rate. We denote this riskless rate of return by r. Thus, an investment of W dollars at time t yields $W(1+r)$ at time $t+1$. The return on the stock is composed of two elements:

(i). Capital gain (loss): The price of the stock is determined collectively by all investors by the law of supply and demand. If an investor holds a stock, any rise (fall) in the price of the stock contributes to an increase (decrease) in the investors' wealth.

(ii). Dividends: The company earns income and distributes dividends. We assume that the firm pays a dividend of D_t per share at time t. We will elaborate on D_t when we discuss the parameters of the model. Thus, the overall rate of return on stock in period t, denoted by H_t, is given by:

$$H_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \tag{1}$$

where P_t is the stock price at time t. In order to decide on the optimal diversification between the risky and the riskless asset, one should consider the ex-ante returns. However, since in practice these returns are generally not available, we assume that the ex-post distribution of returns is employed as an estimate of the ex-ante distribution. In our model investors keep track of the last k returns on the stock, which we call the stock's history. We assume that investors have a bounded recall in that they believe that each of the last k history elements at time t $H_j, \quad j = t, t-1, ..., t-k+1$ has an equal probability of $1/k$ to reoccur in the next time period $(t+1)$. The bounded recall framework has been employed in other game theory analyses [16-18]. Thus, as in real life, investors are confronted with an investment decision where the outcome is uncertain. According to the standart theory of investment under uncertainty investors derive "well being" or "utility" from their wealth. Each investor is characterized by a utility function, $U(W)$, reflecting his/her personal preference. In a situation with uncertainty the objective of each investor is to maximize the expected value of his/her utility [19]. In the present work we assume the same utility function for all investors, and we take this function to be $\ln W$, which is very common in the economic literiture [20-22]. Investors divide their money between the two investment options in the optimal way which maximizes their expected utility. We will elaborate on this point below.

1.1 The Dynamics

To illustrate the dynamics of our model consider the state of the market at some arbitrary time t. We denote the price of the stock at this time by P_t. The stock's history at this time is a set of the last k returns on the stock, $H_j,\ \ j = t, t-1, ..., t-k+1$. We denote the wealth of the ith investor at time t by $W_t(i)$, and the number of shares held by this investor by $N_t(i)$. Now, let us see what happens at the next trade point, time $t+1$.

Income Gain

First, note that the investor accumulates wealth in the interval between time t and time $t+1$. He/she receives $N_t(i)D_t$ in dividends and $(W_t(i) - N_t(i)P_t)r$ in interest. ($W_t(i) - N_t(i)P_t$ is the money held in bonds as $W_t(i)$ is the total wealth, and $N_t(i)P_t$ is the wealth held in stocks). Thus, before the trade at time $t+1$, the wealth of investor i is:

$$W_t(i) + N_t(i)D_t + (W_t(i) - N_t(i)P_t)r. \tag{2}$$

During the interval between time t and time $t+1$ there is no trade, therefore the share price does not change and there is no capital gain or loss. However, at the next trade, at time $t+1$, capital gain or loss can occur, as explained below.

The Demand Function for Stocks

We derive the aggregate demand function for various hypothetical prices P_h, and based on it we find $P_{h^*} = P_{t+1}$, the equilibrium price at time $t+1$. Suppose that at the trade at time $t+1$, the price of the stock is set at a hypothetical price P_h. How many shares will investor i want to hold at this price? First, let us observe that immediately after the trade the wealth of investor i will change by the amount $N_t(i)(P_h - P_t)$ due to capital gain (or loss). Note that there is capital gain or loss only on the $N_t(i)$ shares held before the trade, and not on shares bought or sold at the time $t+1$ trade. Thus, if the hypothetical price is P_h, the hypothetical wealth of investor i after the $t+1$ trade, $W_h(i)$, will be:

$$W_h(i) = W_t(i) + N_t(i)D_t + (W_t(i) - N_t(i)P_t)r + N_t(i)(P_h - P_t) \tag{3}$$

where the first three terms are from eq. (2). The investor has to decide at time $t+1$ how to invest this wealth. He/she will attempt to maximize his/her expected utility at the next period, time $t+2$. As explained before, the ex-post distribution of returns is employed as an estimate for the ex-ante distribution. If investor i invests at time $t+1$ a proportion $X(i)$ of his/her wealth in the stock, his/her expected utility at time $t+2$ will be given by:

$$EU\Big(X(i)\Big) = 1/k \sum_{j=t}^{t-k+1} \ln\Bigg[\Big(1 - X(i)\Big)W_h(i)(1+r) + X(i)W_h(i)\Big(1+H_j\Big)\Bigg]$$

where the first term in the square brackets is the bond's contribution to his/her wealth and the second term is the stock's contribution. The investor will choose the investment proportion, $X(i)$, that maximizes his/her expected utility [1]. We denote this optimal proportion (which we find numerically) by $X_h(i)$.

The amount of wealth that investor i will hold in stocks at the hypothetical price P_h is given by $X_h(i)W_h(i)$. Therefore, the number of shares that investor i will want to hold at the hypothetical price P_h will be:

$$N_h(i, P_h) = \frac{X_h(i)W_h(i)}{P_h}. \tag{4}$$

This constitutes the personal demand curve of investor i. Summing the personal demand functions of all investors, we obtain the following collective demand function:

$$N_h(P_h) = \sum_i N_h(i, P_h) \tag{5}$$

Market Clearance

As the number of shares in the market, denoted by N, is assumed to be fixed, the collective demand function determines the equilibrium price P_h^*. P_h^* is given by the intersection point of the aggregate demand function and the supply function, which is a vertical line. Thus, the equilibrium price of the stock at time $t+1$, denoted by P_{t+1}, will be P_h^* .

[1] As now borrowing or shortselling is allowed, we have $0 \leq X \leq 1$. However, we introduce a constraint asserting that $X < b < 1$ where b is very close to 1, (e.g., 0.99). Thus, the assumption is that even if the pure mathematical solution advocates 100% investment in stocks, to guarantee some money for emergency needs, investors will not invest more than b in the stock, and they will keep some money in (riskless) bonds. If we introduce borrowing, we will still have an upper bound on X, set by the bank. In this case we would have $b > 1$.

History Update

The new stock price, P_{t+1} and dividend D_{t+1}, give us a new return on the stock, H_{t+1}:

$$H_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t}.$$

We update the stock's history by including this most recent return, and eliminating the oldest return H_{t-k+1} from the history. This completes one time cycle. By repeating this cycle, we simulate the evolution of the stock market through time.

Notations

r - riskless interest rate

P_t - price of stock at time t

D_t - dividend at time t

H_t - rate of return of the stock at time t

k - memory span

$W_t(i)$ - wealth of investor i at time t

$N_t(i)$ - number of shares held by investor i at time t

$X(i)$ - optimal proportion of investment for investor i

P_h - hypothetical price

$W_h(i)$ - wealth of investor i given the hypothetical price P_h

$N_h(i)$ - demand for stocks of investor i given the hypothetical price P_h

1.2 Deviations From Rationality

The model described so far is deterministic. The decision making process is conducted by maximizing expected utility. It is a bounded rational, predictable decision making. In more realistic situations, investors are influenced by many factors other than rational utility maximization [23]. The net effect of a large number of uncorrelated random influences is a normally distributed random influence or "noise". Hence, we take into account all the unknown factors influencing decision making by adding a normal random variable to the optimal investment proportion. To be more specific, we replace $X(i)$

with $X^*(i)$ where

$$X^*(i) = X(i) + \epsilon(i) \tag{6}$$

and $\epsilon(i)$ is drawn at random from a normal distribution with standard deviation σ. We should emphasize that $X(i)$ is the same for all investors, but $X^*(i)$ is not, because $\epsilon(i)$ is drawn separately for each investor.

1.3 The Parameters

In the simulations described in this paper, we choose the time period between each trade to be one day. Accordingly, we choose the rest of the parameters realistically. We take the daily interest rate to be 0.01% (yeilding a 3.7% annual interest rate). The initial history, consists of a discrete distribution of returns with a mean of 0.0100025% and a standard deviation of 0.0125%. With these parameters, the investment proportion in the risky asset in the first round is about 50%, thus the bond and stock are more or less compatible in the initial stage. The number of investors is 100 and the number of outstanding shares is 10,000. The initial wealth of each investor is $1,000. The initial share price is $4.00. The initial dividend is taken to be $0.004 . We increase the dividend by 0.015% daily, to represent firm growth. This growth rate yields an annual growth rate of 5.6%, which is close to the long run average dividend growth rate of the S&P. We should stress that our results are general and that there was no fine-tuning of the parameters. The main features of the long run dynamics are insensitive to the initial conditions.

2. Homogeneous Investors

The investors of our model are characterized by their utility functions and their memory spans. In all of the simulations presented in this paper, we assume a logarithmic utility function for all investors, so investors may differ only with regard to their memory spans (and their investor-specific noise). The first case we study is that of homogeneous investors. Figure 1 depicts the price of the stock as a function of time, in a market of investors all having a memory of the last 10 returns on the stock (k=10).

The stock price alternates regularly between two very different price levels. The explanation for this dynamics is as follows :

The rate of return on the stock from the first trade is higher than the oldest remembered return that is deleted from the history. This creates a distribution of returns that is "better" than the initial history, where "better" means that investors are willing to increase their investment proportion in the stock. When investors increase their investment in the stock the stock price goes up, generating an even higher return. This positive feedback stops only when investors reach the maximum investment proportion (99%), and can no longer increase their investment proportion in the stock. This happens at point **b** (Figure 1).

Once the price reaches the high level, the returns on the stock are not very high, because the dividend is now very small compared with the high price. In [15] it was shown that in the abscence of noise the returns on the stock at this plateau converge to the constant growth rate of the dividend, which is just slightly higher than the riskless interest rate. In other words, in the absence of noise the price remains almost constant, growing only because of the interest payed on the bond (more money entering the system and being invested in the stock). When there is some noise in the system the price fluctuates a little around the high level, because of fluctuations in the investment proportions. These fluctuations generate some negative returns (on a downward fluctuation) and some high returns (when the price goes back up).

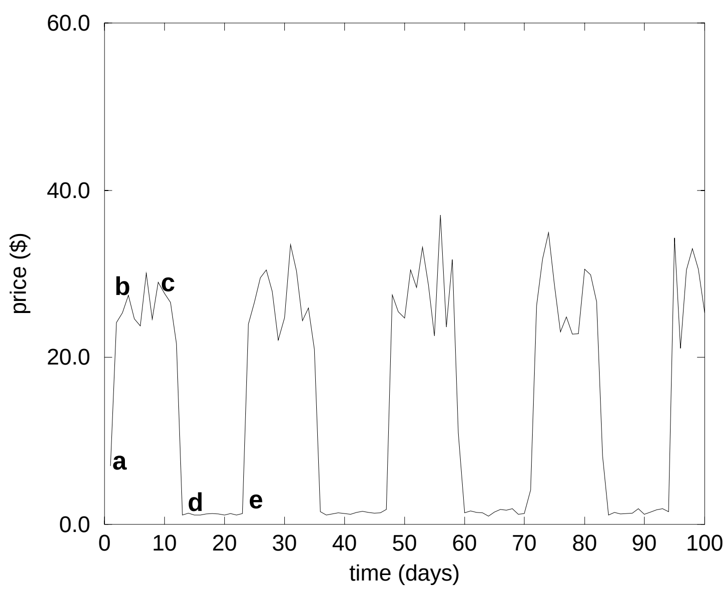

Fig. 1. — Stock price as a function of time in a market of homogeneous investors with a memory span of 10 days.

One might suspect that a large downward fluctuation might trigger a reverse positive feedback effect, where investment proportions will decrease, the price will drop, generating further negative retuns and so on - a crash. This can happen, but only after the sharp price boom (**a-b**), which generates an extremely high return, is forgotten. And, indeed, this is exactly what happens at point **c**. Since it takes 10 days to forget the boom, the high price plateus are a bit longer than the memory span (10 days to forget the boom + a few more days untill a large enough negative fluctuation occurs).

The crash (**c-d**) generates a disastarous return and, untill it is forgotten, investment proportions and hence the price remain very low. When the price is low, the dividend becomes significant and the returns on the stock are relatively high. Once the crash has been forgotten (**e**), all the returns that are remembered are high, and the price jumps back up. Thus, the low price plateaus are 10 days long. This completes one cycle, which is repeated throughout the run.

Figure 2 shows the Fourier transform of this run. As expected the main peak is at a frequency f_0 bit lower than 0.05 (0.0415), corrosponding to a cycle length a bit higher that 20 days (24.1 days). The other peaks : $3f_0, 5f_0, ...$ are due to the fact that the signal resembels a square wave, rathar than a sinusuidal wave.

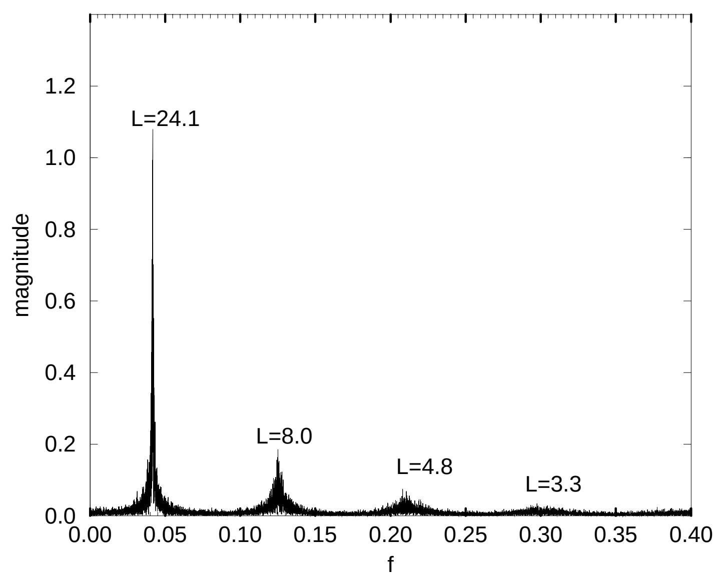

Fig. 2. — Fourier transform of the price in a market of homogeneous investors with a memory span of 10 days.

The dynamics of a market with homogeneous investors is obviously very unrealistic. The booms and crashes are not only gigantic but also periodic and therefore easaly predictable. For a more detailed account of the homogeneous population dynamics see [13-15]. In the next section we study the dynamics of a market with two different investor populations.

3. Two Investor Populations

Most models of the stock market assume that the entire investor population can be represented by a single "average" investor. This assumption, which is analougous to the mean field approximation, is made for the sake of simplifying analytical treatment. In this section we show that by making this approximation one loses the essence of the dynamics. As will be shown below, it is precisely the nonlinear interaction between different investor populations that makes the dynamics interesting and complex. In this section we study the most simple nontrivial case, the case of two investor populations. This is still a very simplified case, but it gives the flavour of the dynamics of more realistic and complicated systems.

It turns out that the nature of the dynamics of a two-population market is determined by the ratio of the memory spans of the two populations. In order to understand this, consider the case where one population dominates the market and dictates the dynamics. Let us ask ourselves what will happen to a second population in such a market. First of all note that the dominant population (which we denote by population$_0$, having a memory span m_0) is not doing well on average. Since it is this population that dictates the dynamics, by definition, this population "goes with the trend" (which it creates). A boom occurs when population$_0$ buys, so population$_0$ buys at the high price and therefore gains nothing from a boom (profit is made only on stocks held **before** the boom). A crash occurs when population$_0$ sells, and therefore population$_0$ sells at the low price and loses at the crash. Consider the following arguments :

A. If the second population (population$_1$) has a memory span in the range $m_0 < m_1 < 2m_0$ it will do even worse than population$_0$. The reason for this is that population$_1$ will also buy at the high price and sell at the low price (see Figure 3). Population$_1$ will do even worse than population$_0$, because **before** the boom it will hold less stocks than population$_0$, because it will remember not only the crash, but some of the preceding low returns, whereas population$_0$ will remeber only the crash and the following high returns. By a symmetric argument population$_1$ holds more stocks than population$_0$ before a crash.

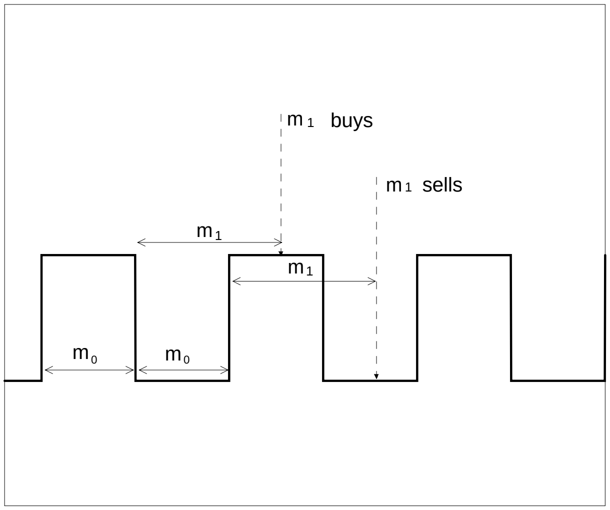

Fig. 3. — Investors with a memory span in the range $m_0 < m_1 < 2m_0$ are doing worse than the dominating population (memory span m_0).

B. If the memory span of population$_1$ is in the range $2m_0 < m_1 < 3m_0$ population$_1$ will be better of than population$_0$. The investors of population$_1$ always remeber one boom and one crash. These are by far the most dominant returns in the memory and will therefore dictate a more or less constant investment proportion, which is better than what population$_0$ is doing. For $2nm_0 < m_1 < (2n+1)m_0$, a similar argument holds. The bigger n, the more stable the investment proportions of population$_1$.

C. Investors with a memory span in the range $3m_0 < m_1 < 4m_0$ will always have in memory three dramatic events (either two booms and a crash or a boom and two crashes). They will buy at the high price and sell at the low price, but they will do so more moderately than population$_0$. Two booms and one crash in memory generate a lower investment proportion than just one boom. As a result this population will do a bit better than population$_0$, but not as good as the population holding a more or less constant investment proportion $(2nm_0 < m_1 < (2n+1)m_0)$. The same argument holds for the general case of investors with memory spans in the ranges $(2n+1)m_0 < m_1 < 2nm_0, \quad n > 1$.

D. Finally, the investors that will be best off in this situation are those with a memory span shorter than m_0. They will buy before the boom, holding many stocks before the price increase and therefore making a big profit at the boom, and they will sell before the crash, surviving it without any loss.

To summarize, in a sitution with cycles of length $2m_0$:

A : $m_0 < m_1 < 2m_0$, m_1 is performing very poorly

B : $2nm_0 < m_1 < (2n+1)m_0$, m_1 is doing relatively well

C : $(2n+1)m_0 < m_1 < 2nm_0$, $n > 1$, better than A but worse than B

D : $m_1 < m_0$, m_1 is doing extremely well

The arguments above assume a situation where one population dictates the dynamics and the second population is affected by the dynamics, but does not affect it. This is of course unrealistic, but the above arguments are very helpfull in understanding the more complicated actuall dynamics.

The first two-population case we studied is a market with half of the investors having memory span 10, and the other half with memory span 14. Figure 4 shows the fraction of the wealth of the population with memory 10 out of the total wealth. It is clear that this population quickly takes over the market completely. Figure 5 is the Fourier transform of this run. This Figure is very similar to Figure 2 - the investors with memory 14 do not affect the dynamics at all. Why does memory 10 have such clear dominance over memory 14 ? We know from argument A that when memory 10 dominates, memory 14 is worse off than memory 10. This is also true when memory 14 dominates (argument D). Therefore memory 14 does not have a chance to win - and is completely wiped out by memory 10.

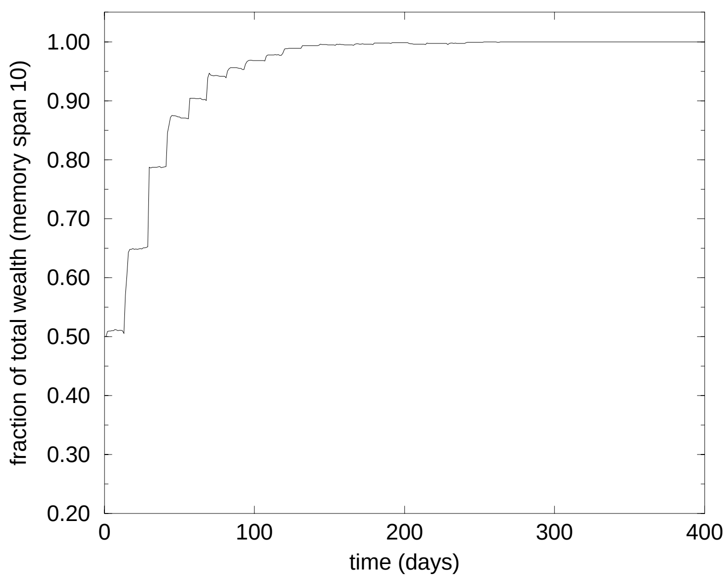

Fig. 4. — Fraction of the wealth of the population with memory 10 out of the total wealth.

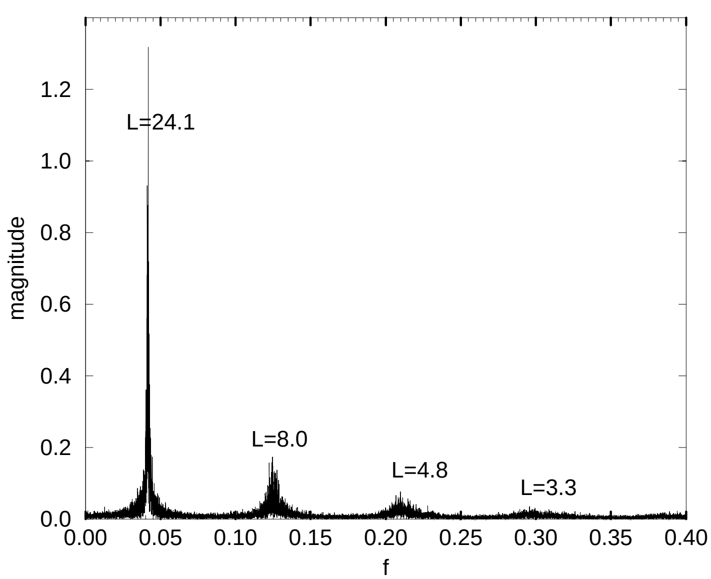

Fig. 5. — Fourier transform of the price in a market with two equal investor populations, memory spans 10 and 14 days

The second case we studied is, again of two equal populations, one with memory 10, and the other with memory 26. In this case memory 26 is better of when memory 10 dominates (argument B) but memory 10 is better of when memory 26 dominates (argument D). It is therefore reasonable that one population can not dominate the other indefinately. Indeed, a look at the fraction of the wealth of the population with memory 10 out of the total wealth reveals alternating eras of dominance (Figure 6). It is interesting that memory 26 dominates most of the time. It is even more interesting that during a large portion of this time the cycles are short, and do not correspond to the cycle length of a bit more than 52 that we would expect of memory 26 dominance. We see that memory 26 begins to dominate the wealth very early in the run, whereas longer cycles corresponding to a memory of 26 begin only around day 2000 (Figure 7). How is it possible that memory 26 dominates, yet the cycles remain short ? This can be to understood by looking at Figure 8. Figure 8 depicts schematically the following exaggerated situation : memory m_0 dominates for a while untill at time t_0 memory m_1 becomes completely dominant and dictates the dynamics. The first boom occurs only at t_1 when investors forget the crash at $t_1 - m_1$. The following crash occurs at t_2 when investors forget the boom at $t_2 - m_1$, and so on. The resulting dynamics is that of nonidentical short cycles with an average length of $2m_1/3$, and resembles very much the short cycles of Figure 7 (one long plateau, followed by two short plateaus). This state of long memory dominance and short cycles ($2m_1/3$) is quite stable and generic and will be encountered again below.

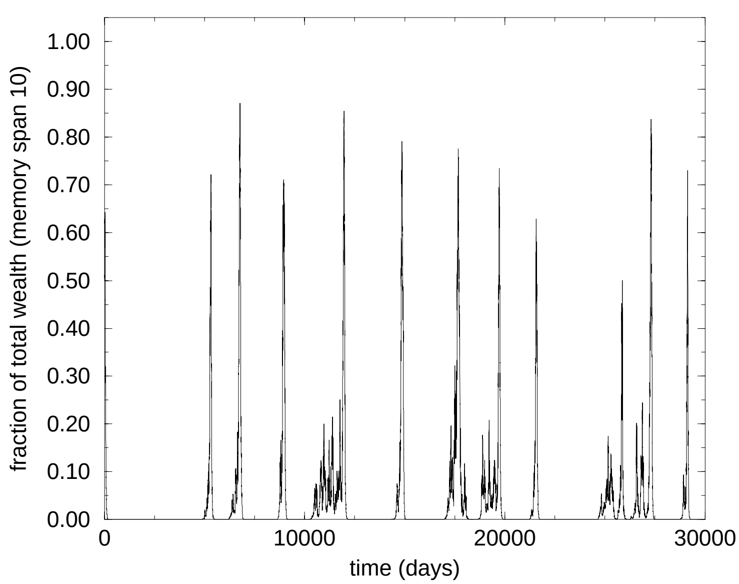

Fig. 6. — Fraction of the wealth of the memory 10 population.

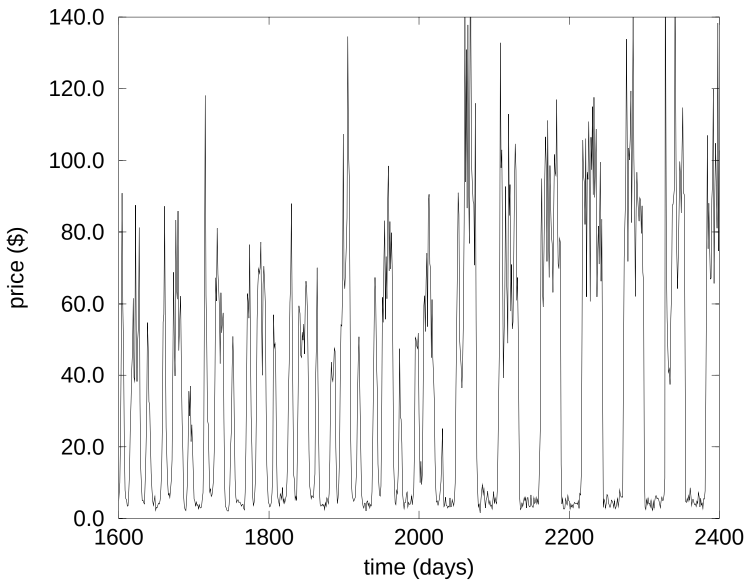

Fig. 7. — Stock price as a function of time in a market with two equal investor populations, memory spans 10 and 26 days .

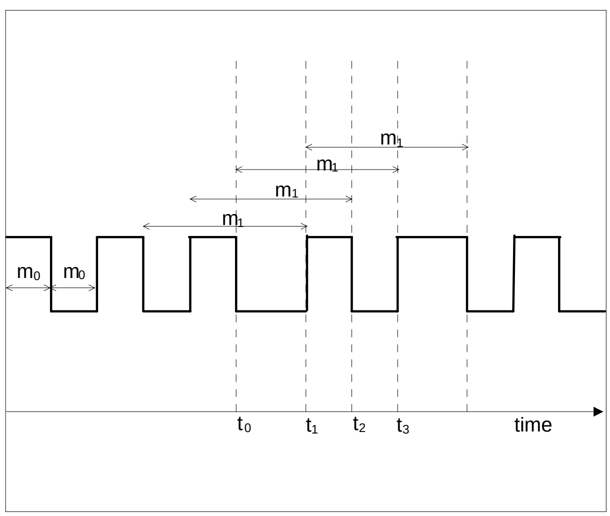

Fig. 8. — Meta-stable state where long memory dominates the wealth yet the cycles are short.memory m_0 dominates untill time t_0 when memory m_1 becomes completely dominant and dictates the dynamics. The first boom occurs at t_1 when investors forget the crash at $t_1 - m_1$. The following crash occurs at t_2 when investors forget the boom at $t_2 - m_1$.

Who is better off in this meta-stable state ? m_1 is the trend maker and can not be doing very well. On the other hand m_0 goes with the trend 4 out of 6 times, and does so more extremely than m_1, which avarages more and therefore maintains a more stable investment proportion (argument C) . It turns out, that at least in this case, memory 26 is better off in this situation (see Figure 6, day 0-2000).

Only when the system gets out of this meta-stable state and enters a phase of long cycles (around day 2000) the shorter memory population begins to gain dominance. Finally, at around day 5300 it gains enough power to dictate the dynamics again (Figure 9). When the cycles are short the memory 10 population quickly loses its dominance (as there is no opposite metastable state with short memory dominance and long cycles). This explains why memory 26 dominates the wealth most of the time, and also the asymmetrical form of the peaks in Figure 6.

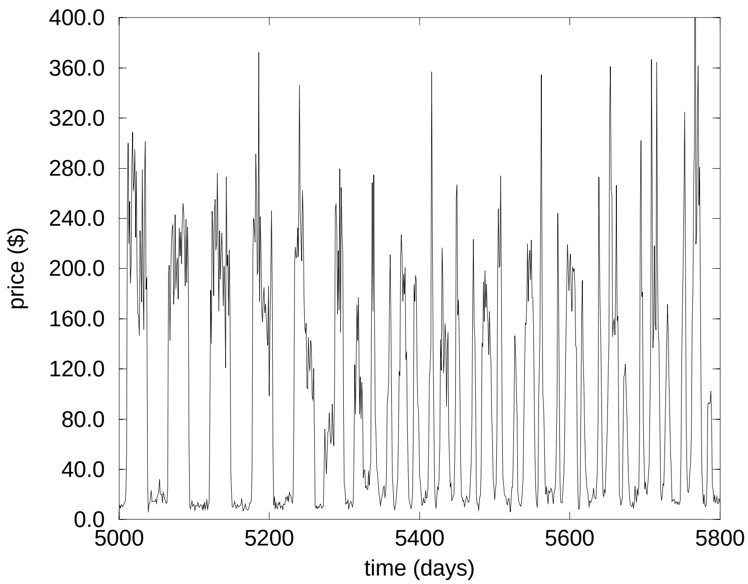

Fig. 9. — Stock price as a function of time in a market with two equal investor populations, memory spans 10 and 26 days .

In the Fourier transform of this run (Figure 10) we see the long cycles corresponding to memory 26. We also see that the short cycles are of length 18.6 days (55.6/3) rather than 24.1 days corresponding to memory 10 (see Figures 2). This in agreement with our analysis of the meta stable state. [2]

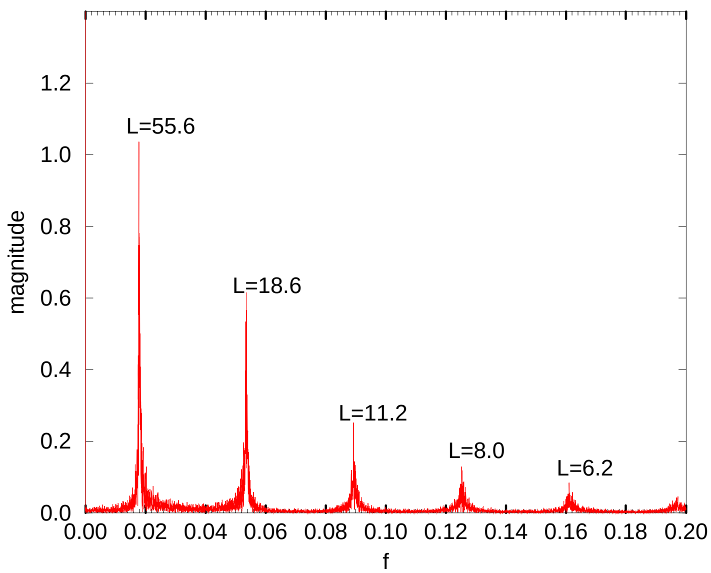

Fig. 10. — Fourier transform of the price in a market with two equal investor populations, memory spans 10 and 26 days .

The last two-population market that we study is that of equal populations with memory spans 10 and 36. Similarly to the 10,26 memory market, the investors with memory 36 are doing better than those with memory 10 when memory 10 dictates the dynamics (argument C), but those with memory 10 are doing better when memory 36 dictates the dynamics (argument D). Hence, we may speculate that again we will find alternating eras of dominance. Figures 11 and 12 show that this is not the case. We see in the Fourier transform that only short cycles are present. Did the memory 10 population take over the market ? Figure 12 tells us that this is not the case. In fact, the memory 36 population dominates about 70-80% of the total wealth throughout the run. This means that again we are seeing a situation where the long memory population dominates wealth but the cycles are

[2] One might suspect that the peak at 18.6 is just the third harmonic of the long cycle, however, the ratio between the third and first harmonics when there are only long cycles present is approximately 0.17 (see Figure 2), whereas here this ratio is approximately 0.58. Also, we see the short cycles directly in Figures 7, 9.

short. The explanation here is very similar to that explaining Figure 8, and again the short cycles are approximately $2m_1/3$. The difference between this case and the 10,26 memory market is that here the market remains stuck in the metastable state. The memory 36 population never gains enough wealth to dictate long cycles. The reason for this is clear, if we remember that m_1 in the range $(2n+1)m_0 < m_1 < 2nm_0$ is doing not as good against m_0 as does m_1 in the range $2nm_0 < m_1 < (2n+1)m_0$, (argument C). Thus, the system remains in this state of "symbiosis" throughout the run.

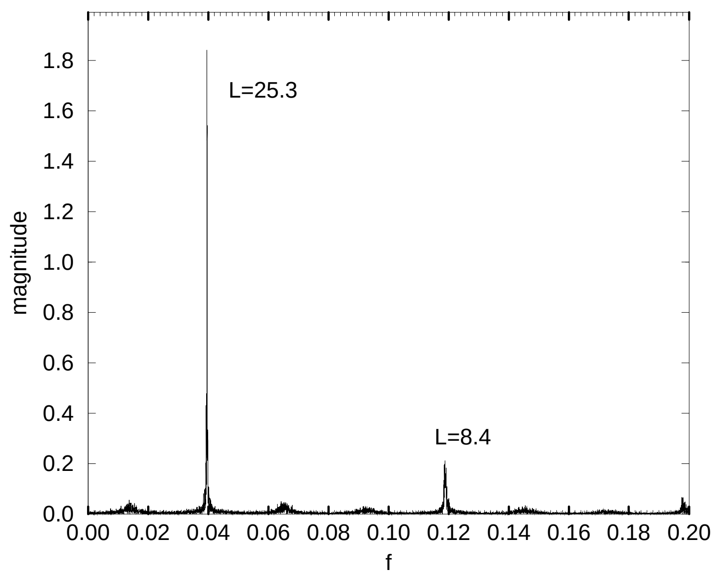

Fig. 11. — Fourier transform of the price in a market with two equal investor populations, memory spans 10 and 36 days .

We have seen very different things that can happen in two population dynamics, depending on the two memory spans. The phenomena of dominance by one population, alternating dominance and symbiosis can be understood in terms of the arguments A-D. Although the dynamics is rich, except for short transitional periods between eras the cycles we obsereve are allways orderly. The time series can be devided into distinct eras where there is a definite cycle length. Within these eras prediction is possible, and therefore the market is unrealistic. In the next section we will see what happens when a third population is introduced into the market.

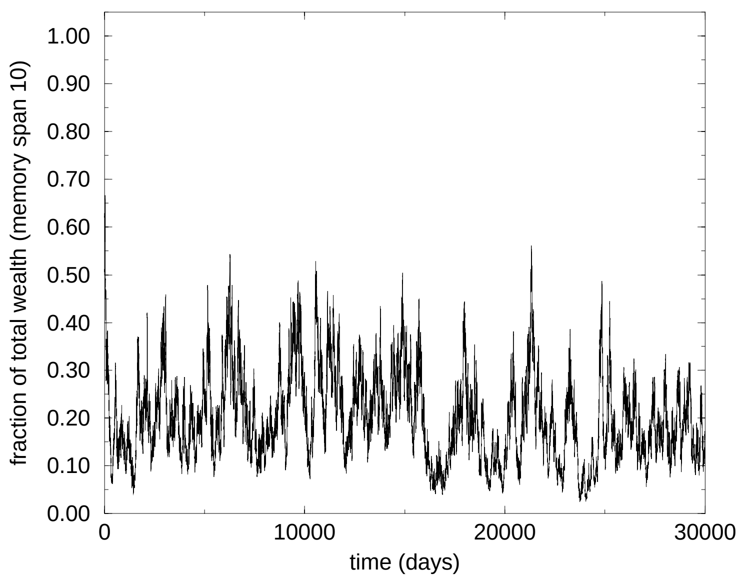

Fig. 12. — Fraction of the wealth of the memory 10 population in a market with two equal investor populations, memory spans 10 and 36 days.

4. Three Investor Populations

One might suspect that the three population dynamics is a natural extention of the two population dynamics. Instead of alternating between two cycle lengths the system may just alternate between the three possible states of dominance . Figure 13 shows that this is not at all the case. This Figure depicts a typical part of the dynamics of a three population market, with memory spans 10,141,256. With the introduction of a third population the system has undergone a qualitative change. There is no specific cycle length describing the time series. Instead, we see a mixture of different time scales - the system has become complex. Prediction becomes very difficult, and in this sense the market is much more realistic. Figure 14 shows the power struggle between the three populations. Figure 15 depicts the Fourier transform of this run. Although the dynamics is complex, it is clear from Figures 14 and 15 that there is an underlying structure, which perhaps may be analyzed by arguments A-D and their generalizations. The dynamics generated by only three investor populations can be extremely complex, even without any external random influences.

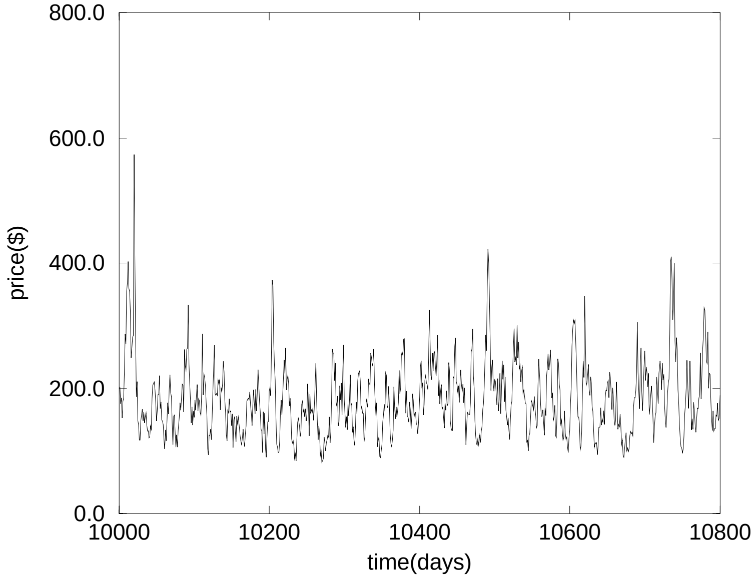

Fig. 13. — Stock price as a function of time in a market with three equal investor populations, memory spans 10,141 and 256 days .

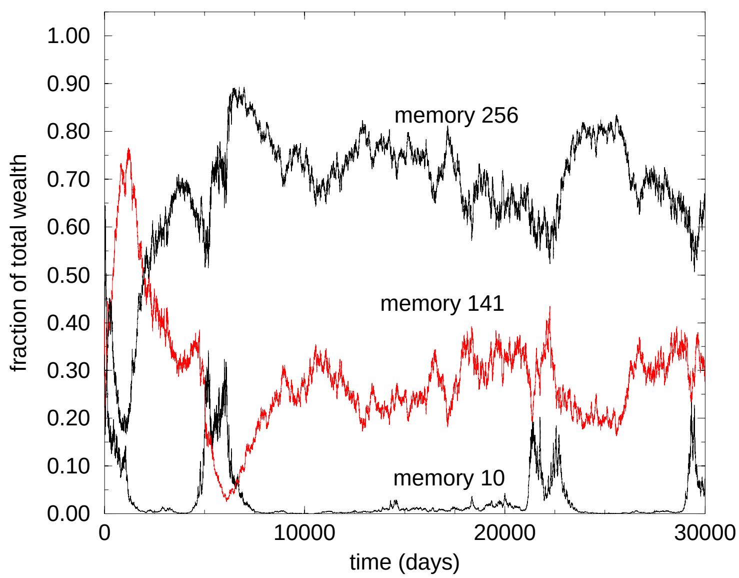

Fig. 14. — Fraction of total wealth as a function of time in a market with three equal investor populations, memory spans 10,141 and 256 days .

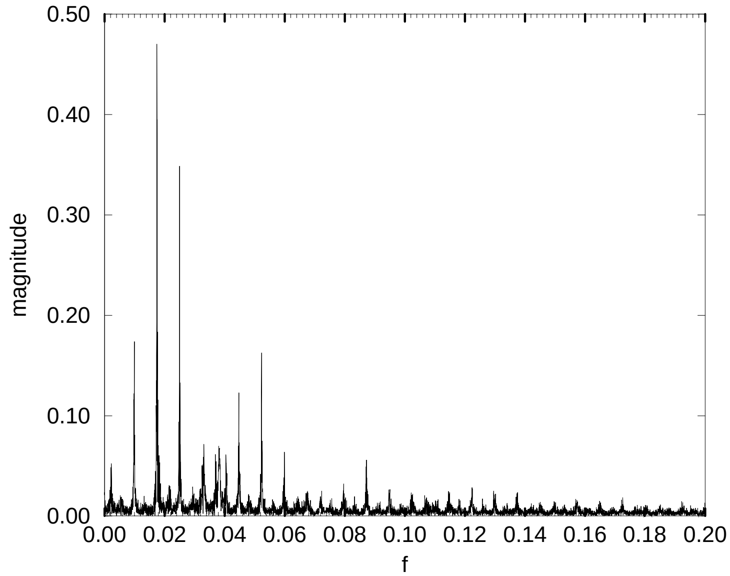

Fig. 15. —Fourier transform of the price in a market with three equal investor populations, memory spans 10,141 and 256 days.

As the number of populations grows, the dynamics becomes more complex and realistic. Figure 16 shows the dynamics of a market with six equal populations with memory spans 10, 36, 141, 193, 256, and 420. One of the effects of introducing more populations is that the amplitude of the fluctuations decreases, and they become more realistic.

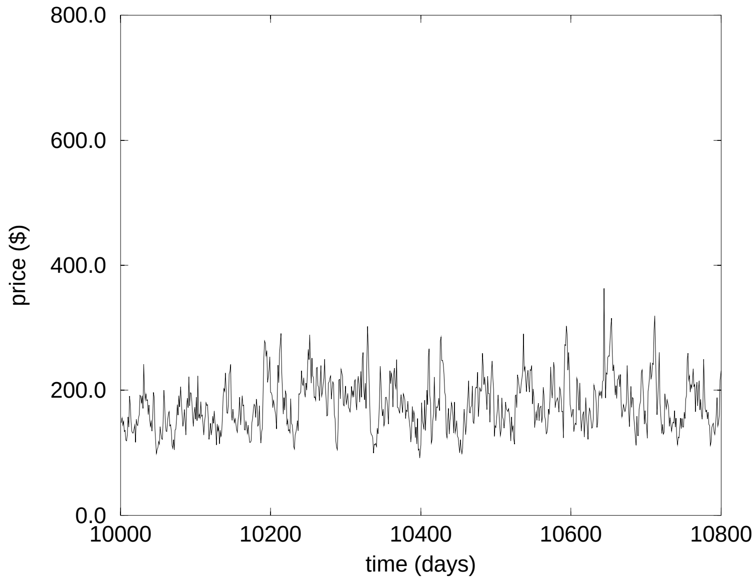

Fig. 16. — Stock price as a function of time in a market with six equal investor populations, memory spans 10, 36,141,193,256 and 420 days .

5. Summary

In this paper we studied a microscopic model of the stock market using simulations. We analyzed the dynamics of this system with one, two, and three investor subgroups, differing only in their memory spans.

When there is only one subgroup the dynamics is ordered and unrealistic. When there are two subgroups we observed phenomena ranging from complete dominance of one population to alternating eras of domination and to "symbiosis". In all these cases, however, the dynamics is ordered, in the sense that the run can always be devided into distinct eras with a dominant cycle.

This is qualitatively and dramatically changed when a third subgroup is introduced. The dynamics of the system becomes complex. The larger the number of investor subgroups, the more realistic and complex the dynamics.

Our results suggest that complexity is an intrinsic property of the stock market. The dynamic and complex behaviour of the market need not be explained as an affect of external random information. It is a natural property of the market, emerging from the strong nonlinear interaction between the different investor subgroups of the market. As such, this complexity can be investigated, rather than being regarded as random noise.

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