Agent Based Computational Finance: Suggested Readings and Early Research *

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Abstract

The use of computer simulated markets with individual adaptive agents in finance is a new, but growing field. This paper explores some of the early work in the area concentrating on a set of some of the earliest papers. Six papers are summarized in detail, along with references to many other pieces of this wide ranging research area. It also covers many of the questions that new researchers will face when getting into the field, and hopefully can serve as a kind of minitutorial for those interested in getting started.

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1 Introduction

Modeling economic markets from the bottom up with large numbers of interacting agents is beginning to show promise as a research methodology that will greatly impact how we think about interactions in economic models. There is already a growing literature attempting to model financial interactions starting from the agent perspective, relying heavily on computational tools to push beyond the restrictions of analytic methods. This survey provides some pointers and synthesis on this new area of research.

Financial markets are an important application for agent based modeling styles. As with many other economic situations, there is a great appeal in starting from the bottom up with simple adaptive, learning agents.[1] Beyond this, financial markets may offer other features which make them even more appealing to agent based modelers. First, issues of price and information aggregation tend to be sharper in financial settings where agent objectives tend to be clearer. Second, financial data is readily available at many different frequencies from annual to minute by minute. Finally, there are continuing developments in the area of experimental financial markets which give carefully controlled environments which can be compared with agent based experiments. Financial markets have also posed many empirical puzzles for standard representative agent models which have been unsatisfactory in explaining them.[2]

Computational agent based models stress interactions, and learning dynamics in groups of traders learning about the relations between prices and market information. The use of heterogeneous agents is certainly not new to finance, and there is a long history to building heterogeneous agent rational expectations models.[3] What is attempted in this current set of computational frameworks is to attack the problem of very complex heterogeneity which leaves the boundary of what can be handled analytically. Traders are made up from a very diverse set of types and behaviors. To make the situation more complex the population of agent types, or the individual behaviors themselves, are allowed to change over time in response to past performance.

Instead of trying to survey the literature, this paper will emphasize the results in six early papers. These are chosen both because they are some of the earliest papers, and span an important realm of work in artificial financial markets. In short, they are some of the most important papers that new researchers should be aware of when they are just starting in this field. The next section will go through these brief

[1] The best place for information on this is the web site maintained by Leigh Tesfatsion at Iowa State, http://www.econ.iastate.edu/tesfatsi/ace.htm.

[2] This list of puzzles includes rejections of the standard representative agent model, and the equity premium puzzle (Hansen & Singleton 1983, Mehra & Prescott 1985, Hansen & Jagannathan 1990), volatility persistence (Bollerslev, Chou, Jayaraman & Kroner 1990), and the use and performance of technical trading rules (Brock, Lakonishok & LeBaron 1992, Levich & Thomas 1993, LeBaron 1998, Sweeney 1986). Kocherlakota (1996) is a recent survey of the large body of research on the equity premium.

[3] Some of the origins of this are contained in Grossman (1976) and Grossman & Stiglitz (1980).

paper summaries. Any good survey should also give some of the key pointers to other papers in the area, and it should be emphasized that the limitation in discussing only six papers was a severe binding constraint. Some discussion and references will be made to the many other papers in this field in section three. While the purpose of this paper was to concentrate on computational models, there are analytic approaches that are closely related. Leaving out discussion of these papers would make this summary incomplete. The third section also discusses a few of these papers along with the interactions between computational and analytic approaches. The fourth section expands on the general survey of the paper by covering some of the issues that need to be considered by anyone setting up their own computational experiments. Since this area is new, there are still a large number of questions about market and agent design that remain unanswered. A few of these issues will be discussed along with some suggestions for the future. The final section provides a short conclusion.

2 Paper summaries

2.1 Simple agent benchmarks

Lettau (1997) is probably the best place to start for anyone thinking about constructing artificial agents. This paper implements many of the ideas of evolution and learning in a population of traders in very simple setting which provides a useful benchmark. Many of the issues brought up in this paper remain important in all work on computational agents, but they show up more clearly in this very simple market.

In his framework agents decide how much of a risky asset to purchase. The asset is sold at a price, p, and issues a random dividend next period paying, d, drawn from a gaussian distribution. The agents must chose between this risky asset and a risk free bond paying zero interest. Two big simplifications greatly clarify the situation. First, the price is given exogenously. This assumption may appear to go against the entire concept of building artificial markets, but it allows Lettau to concentrate on the agents' behavior. Since the actual behavior of evolutionary agents is not well understood, this seems like a very good idea. Second, the agents are assumed to have myopic constant absolute risk aversion preferences. They maximize

$$U(w) = E(-e^{-\gamma w}) \tag{1}$$

$$w = s(d - p), \tag{2}$$

where s is the number of shares held of the risky asset, the agent's only choice variable.

For a fixed distribution of d it is well known that the optimal solution for s will be a linear function of the price, p, and the mean dividend $\bar{d}$.

$$s^* = \alpha^*(\bar{d} - p) \tag{3}$$

Lettau is interested in how close evolutionary learning mechanisms get to this optimal solution.[4]

To do this he implements a genetic algorithm (GA) which is a common tool in many computational learning models. The GA was developed by Holland (1975), and is a biologically inspired learning method. A population of candidate solutions is evolved over time by keeping the best, removing the worst, and adding new rules through mutation and crossover. Mutation takes old rules and modifies them slightly, and crossover takes two good rules and combines them into a new rule.[5] It is clear that a complete solution is described by α, and a population of these can be maintained.

In the early era of GA's researchers concentrated on implementations using bitstrings.[6] Given that the solutions here require real values, some kind of translation is needed. Lettau follows a common convention of using the base two integer representation for a fraction between some max/min range,

$$\alpha = MIN + (MAX - MIN)\frac{\sum_{j=1}^{L} \mu_j 2^{j-1}}{2^L - 1}, \tag{4}$$

where μ is the bitstring for a strategy. Bitstrings are now mutated by flipping randomly chosen bits, and crossover proceeds by choosing a splitting position in μ at random, and getting bits to the left of this position from one parent, and right from the other.

This framework is not without some controversy. Learning and evolution takes place in a space that is somewhat disconnected from the real one. For example, mutation is supposed to be concerned with a small change in a behavioral rule. However, for this type of mapping one might flip a bit that could change holdings by a large amount. Many papers in the GA area have moved to simply using real valued representations, and appropriately defined operators rather than relying on mapping parameters. Lettau (and others) are following the procedure of the early papers from computer science, and I believe most of the results would not be sensitive to changing this, but it would be an interesting experiment.

[4] In earlier working versions of this paper Lettau considered more complicated functional forms for $s(\bar{d}, p)$ in the learning procedure.

[5] The GA is at the heart of many of the models discussed here. It is really part of a family of evolutionary algorithms which include evolutionary programming, evolutionary strategies, and genetic programming. As these various methods have been modified, the distinctions across techniques have become blurred. Fogel (1995) and Bäck (1996) give overviews of some of these other evolutionary methods.

[6] See Goldberg (1989) for summaries.

A second critical parameter for Lettau is the choice of sample length to use for the determination of fitness. A candidate rule, i, is evaluated by how well it performs over a set of S experiments,

$$V_i = \sum_{j=1}^{S} U_i(w_{i,j}), \tag{5}$$

where $w_{i,j}$ is the wealth obtained by rule i in the jth experiment. Since fitness is a random variable, the number of trials used to determine expected utility is critical. Extending S to infinity gives increasingly more precise estimates of rule fitness.

In general, Lettau's results show that the GA is able to learn the optimum portfolio weight. However, there is an interesting bias. He finds that the optimal rules have a value for α which is greater than the optimal α^*. This bias implies that they generally will be holding more of the stock than their preferences would prescribe. This is because agents are choosing

$$\alpha^{**} = \arg\max_{\alpha_i} \sum_{j=1}^{S} U_{\alpha_i}(w_j), \tag{6}$$

and $E(\alpha^{**}) \neq \alpha^*$. Intuitively, the reason is quite simple. Over any finite sample, S, there will be a set of rules that do well because they faced a favorable draw of dividends. Those that took risks and were lucky will end up at the top of the fitness heap. Those that took risks and were unlucky will be at the bottom, and the conservative rules will be in the middle. As the GA evolves, this continues to suggest a selection pressure for lucky, but not necessarily skillful strategies. Pushing S to infinity exposes agents to more trials, and the chances of performing well do to chance go to zero. Lettau shows that for very large S, the bias does indeed get close to zero as expected.

This last feature of the GA in noisy environments carries over into many of the other papers considered here. Deciding on fitness values to evolve over is a critical question, and almost all of the papers used here are forced to make some decision about how far back their agents should look, and how much data they should look at. This issue is probably not one that is specific to computerized agents, and is one that is not considered enough for real life behavior in financial markets.[7]

[7]See Benink & Bossaerts (1996) for a version of this issue in an analytic framework.

2.2 Zero Intelligence traders

The next paper may not really be a move up the ladder of intelligence in terms of of agents, but it is another crucial early benchmark paper with computational agents. After observing the behavior of many real trading experiments in laboratories, Gode & Sunder (1993) were interested in just how much "intelligence" was necessary to generate the results they were seeing. They ran a computer experiment with agents who not only do not learn, but are almost completely random in their behavior.

The framework they are interested in is a double auction market similar to those used in many laboratory experiments. Buyers can enter bids for an asset, or raise existing bids. Sellers can enter offers, or lower existing offers. A match or cross of bids and offers implements a transaction. Value is induced as in Smith (1976), where buyer i is given a redemption value of v_i for the asset and therefore a profit of $v_i - p_i$ from a purchase at price p_i. Sellers are given a cost to obtain the asset of c_i, and therefore a profit of $p_i - c_i$. Note, sellers and buyers do not overlap in these simple market settings. It is easy to evaluate efficiency of the market by looking at the profits earned relative to the maximum possible. This is also the total consumer and producer surplus in these markets.

The traders behavior is basically random, issuing random bids and offers distributed over a predefined range. The authors implement one important restriction on their traders. They perform experiments where the trader is simply restricted to his/her budget constraint. For example, a buyer would not bid more than what the asset is worth in redemption value, and a seller will not offer below cost. Beyond this restriction they continue to bid and offer randomly. Their results show that this budget constraint is critical.

Markets with human traders in the experiments are quite well behaved, often converging to the equilibrium price after only a few rounds. The random computer traders that are not subject to budget constraints behave, as expected, completely randomly. There is no convergence, and transaction prices are very volatile. The budget constrained traders behave quite differently, exhibiting a calmer price series which is close to equilibrium. Market efficiency tests support these results by showing that the constrained traders allocate the assets at over 97% efficiency in most cases which is very close to that for humans. The completely random traders fall well back with efficiencies ranging from 50% to 100%.[8]

The message of this paper is critical to researchers building artificial markets. After a set of agents is carefully constructed, it may be that subject to the constraints of the market they may be indistinguishable from ZI traders. This means that researchers need to be very cautious about which features are do to learning

[8] Recent work by Cliff & Bruten (1997) shows that the convergence to the equilibrium price may not occur in all market situations. The relative slopes of the supply and demand curves are critical in determining how close the ZI agents get to the equilibrium price.

and adaptation, and which are coming from the structure of the market itself.

2.3 Foreign exchange markets and experiments

The following papers are more extensive than the first two in that they are attempting to simulate more complicated market structures. Arifovic (1996) considers a general equilibrium foreign exchange market in the spirit of Kareken & Wallace (1981). A crucial aspect of this model is that it contains infinitely many equilibria because it is under specified in price space. Learning dynamics are often suggested as a means for seeing if the economy will dynamically select one of these equilibrium prices (Sargent 1993).

The structure of the economy is based on a simple overlapping generations economy where two period agents solve the following problem.

$$\begin{aligned} & \max_{c_{t,t},c_{t,t+1}} \log c_{t,t} + \log c_{t,t+1} \\ \text{st.} \qquad & c_{t,t} \leq w_1 - \frac{m_{1,t}}{p_{1,t}} - \frac{m_{2,t}}{p_{2,t}} \\ & c_{t,t+1} \leq w_2 + \frac{m_{1,t}}{p_{1,t+1}} + \frac{m_{2,t}}{p_{2,t+1}} \end{aligned}$$

The amounts $m_{1,t}$ and $m_{2,t}$ are holdings in the two currencies which are the only method for saving from period 1 to 2, and can both be used to purchase the one consumption good. $p_{i,t}$ is the price level of currency i in period t. Consumption, $c_{m,n}$ is for generation m at time n. The exchange rate at time t is given by,

$$e_t = \frac{p_{1,t}}{p_{2,t}} \tag{7}$$

Finally, when there is no uncertainty the return on the two currencies must be equal,

$$R_t = \frac{p_{1,t}}{p_{1,t+1}} = \frac{p_{2,t}}{p_{2,t+1}}. \tag{8}$$

It is easy to show from the agent's maximization problem that the savings demand of the young agents at t is

$$s_t = \frac{m_{1,t}}{p_{1,t}} + \frac{m_{2,t}}{p_{2,t}} = \frac{1}{2}(w_1 - w_2\frac{1}{R_t}) \tag{9}$$

Given the returns on the currencies are equal, the agent is actually indifferent between which currency should be used for savings. Aggregate currency amounts are assumed to be held constant for each country.

The basic indeterminacy in the model shown by Kareken & Wallace (1981) is that if there is a monetary equilibrium where savings demand and money supplies are equal for an exchange rate, e, then there exists an equilibrium for all exchange rates $(0, \infty)$. This is fairly easy to see intuitively. If there is an equilibrium for a price sequence and exchange rate $(p_{1,t}, p_{2,t}, e)$ then for another e value, $\hat{e}$, it is very easy to find a sequence $(\hat{p}_{1,t}, \hat{p}_{2,t}, \hat{e})$ which maintains the monetary equilibrium.[9] The key feature that allows one to do this is the equivalence between the two currencies as savings instruments. This turns out to be important for learning dynamics as well as multiple equilibria.

Arifovic (1996) performs some experiments on this foreign exchange model in the laboratory with human subjects. The results give exchange rate series which do not settle to any equilibrium. However, the first period consumption series does appear to settle down to a constant value. It is interesting that in stochastic learning models used by Sargent (1993) the exchange rate does eventually converge to a constant value, although the actual level depends on the starting values.

Arifovic (1996) constructs a genetic algorithm learning environment to test learning in this foreign exchange setting. The GA encodes the complete set of choice variables to agents, current consumption and the fraction of savings in each currency, as a bitstring. Following standard GA techniques a population of traders is evolved. This setup differs in one important fashion from Lettau (1997) in that the population is the market of active traders, and their consumption and portfolio decisions endogenously determine the price levels in the market. The population is evolved according to realized utility levels, and fitter traders are modified with crossover and mutation operators in a standard GA procedure. One interesting modification added to the population dynamics is called the election operator. This new technique only allows parents to be replaced by their offspring if the offspring are at least as good as the parents on a test run using the current prices from the latest market. This keeps really bad offspring from entering the population, and helps to keep the population moving in the right direction.[10]

The results of the GA learning procedure are similar to those in the experiments. The exchange rate fails to settle to any constant value, but the first period consumption level is quite stable.[11] There is a nice intuitive explanation for why this is happening, and it is related to the structure of the indeterminacy of the model. In equilibrium agents are indifferent between holding either of the two currencies. At the reproductive stage, the GA will produce candidate agents some of whom will chose currency fractions which are different from those in equilibrium. When these agents are tested they will appear equally fit to the other

[9] This is done by maintaining the returns at a constant level R. Then adjust p_1 so that $e = p_1/p_2$. Now move p_1 and p_2 in proportion to each other to bring total real balance supply into alignment with total real balance demand. See Arifovic (1996) for more details.

[10] It has a similar counterpart in $\mu + \lambda$ selection used in evolutionary strategies, (Bäck 1996).

[11] Arifovic & Gencay (forthcoming 1998) show that the exchange rate series from this model are chaotic.

agents since in equilibrium the returns on the two currencies are the same. However, once these agents are added to the population the prices will have to change to adjust to the new currency's demands. The system will then move away from the equilibrium, as this will probably move the two returns out of alignment. The basic result is that because agents are indifferent between currency holdings, the equilibrium is subject to invasion by nonequilibrium agents, and is therefore unstable under GA based learning.

This paper introduces several important issues for artificial markets. First, it is considering the equilibrium in a general equilibrium setting with endogenous price formation. Second, it compares the learning dynamics to results from actual experimental markets as in Gode & Sunder (1993). Finally, it shows an interesting feature of GA/agent based learning which appears to replicate certain features from the experiments which can't be replicated in other learning environments.

2.4 Costly information and learning

The next paper focuses on ideas of uncertainty and information in financial markets. Using the framework of Grossman & Stiglitz (1980), Routledge (1994) implements a version of their model with GA based learning agents. This paper takes us up another level of complexity in terms of model structure, and only a short sketch of its details will be provided here.

The approach is based on a repeated one shot version of a portfolio decision problem with a costly information signal that agents can decide to purchase. The dividend payout is given by

$$d = \beta_0 + \beta_1 y + \epsilon \tag{10}$$

where y is the signal that can be purchased for a given cost, c. Agents are interested in maximizing expected one period utility given by,

$$E(-e^{-\gamma w_1}|\Omega) \tag{11}$$

$$\text{s.t.} \qquad w_1 = w_0 - \theta c + x(d - p), \tag{12}$$

with x being the number of shares of the risky asset. There is a risk free asset in zero net supply with zero interest. θ is 1 for informed agents and 0 for uninformed. The expectation is conditioned on available information. For the informed agents, it is price and the signal, y. For the uninformed, it is based on price alone. With multivariate normality this leads to a demand for the risky asset of,

$$x = \frac{E(d|\Omega) - p}{\gamma V(d|\Omega)}. \tag{13}$$

This demand is set equal to a noisy asset supply which represents noise trading, and keeps the underlying signal from being fully revealed.

Learning takes place as the agents try to convert their information into forecasts of the dividend payout. The informed build forecasts using the signal alone since the dividend payout is conditioned only on this,

$$E^n(d|y) = \beta_0^{i,n} + \beta_1^{i,n} y. \tag{14}$$

The uninformed base their predictions on their only piece of information, the price.

$$E^n(d|y) = \beta_0^{u,n} + \beta_1^{u,n} p. \tag{15}$$

In these two equations, I and U are the set of informed and uninformed traders, respectively. Finally, to keep the model tractable, the conditional variances for each informed and uniformed agent are assumed to be, $v^{i,n}$ and $v^{u,n}$. Each instance of a trading agent carries with it a vector of parameters which describes its learning state,

$$(\theta_n, \beta_0^{i,n}, \beta_1^{i,n}, v^{i,n}, \beta_0^{u,n}, \beta_1^{u,n}, v^{u,n}). \tag{16}$$

For a given configuration of agents with a fraction λ purchasing the signal the equilibrium price can be determined. This is done by first setting aggregate demand for the risky asset equal to aggregate supply,

$$\sum_{n \in I} \frac{\beta_0^{i,n} + \beta_1^{i,n} y - P}{\gamma v^{i,n}} + \sum_{n \in U} \frac{\beta_0^{u,n} + \beta_1^{u,n} P - P}{\gamma v^{u,n}} = Ne. \tag{17}$$

Now define,

$$T^I = \frac{1}{\lambda N} \sum_{n \in I} \frac{1}{\gamma v^{i,n}} \tag{18}$$

and,

$$T^U = \frac{1}{(1-\lambda) N} \sum_{n \in U} \frac{1}{\gamma v^{u,n}}. \tag{19}$$

These are the average effective risk tolerances for informed and uniformed agents. Now use these to define aggregate β's for $j = 0, 1$.

$$\beta_j^I = \frac{1}{T^I \lambda N} \sum_{n \in I} \frac{\beta_j^{i,n}}{\gamma v^{i,n}} \tag{20}$$

$$\beta_j^U = \frac{1}{T^U (1-\lambda) N} \sum_{n \in U} \frac{\beta_j^{u,n}}{\gamma v^{u,n}} \tag{21}$$

Now equation 17 can be rewritten as,

$$\lambda T^I (\beta_0^I + \beta_1^I y - P) + (1-\lambda) T^U (\beta_0^U + \beta_1^U P - P) = e \tag{22}$$

which can be easily solved for P.

$$P = \alpha_0 + \alpha_1 y + \alpha_2 (e - \bar{e}) \tag{23}$$

$$\alpha_0 = \frac{\lambda \beta_0^I T^I + (1-\lambda) \beta_0^U T^U - \bar{e}}{\lambda T^I + (1-\lambda) T^U (1 - \beta_1^U)} \tag{24}$$

$$\alpha_1 = \frac{\lambda \beta_1^I T^I}{\lambda T^I + (1-\lambda) T^U (1 - \beta_1^U)} \tag{25}$$

$$\alpha_2 = \frac{-1}{\lambda T^I + (1-\lambda) T^U (1 - \beta_1^U)} \tag{26}$$

A rational expectations equilibrium is a pricing function $P(y)$, and learning parameters, such that that the above forecast parameters are the correct ones for all traders, and the expected utilities of the two types of traders are equal. It is well know that this exists, and how to find it (Grossman & Stiglitz 1980). Routledge (1994) shows that it can be supported through a learning dynamic.

In his GA experiments the traders forecast parameters are coded as bit strings for the genetic algorithm as in the previous papers. Also, the bit strings include a bit which represents whether to purchase a signal or not. A population of traders plays 1000 rounds of the 1 period asset market, and each agent's performance is recorded based on expost utilities. The population is then evolved using the GA with standard crossover and mutation operators.

The market is started at an equilibrium of the model with the intention of checking stability under learning. For some parameters stability is maintained, and the market wobbles only slightly around the equilibrium values. However, for some cases the equilibrium is not stable. In one situation the fraction of informed traders goes from 50 percent, to about 100 percent. There is a very interesting story behind what is going on in this case. At the start the system may wobble a little due to the stochastic learning algorithm. This may add a few more informed agents to the population. Given that there are more informed agents, the uniformed agents' forecast parameters are now wrong, and this increases the relative advantage of being informed. More agents buy the signal, and as this happens the pool of uninformed agents becomes very small. Since they are only a finite population in the simulation their ability to learn is greatly diminished because of their small numbers. This poor learning ability of the uninformed agents further weakens their position, and more agents start purchasing the signal. This continues until almost all of the market is informed.

The key parameter driving this difference is the amount of noise on the supply of shares for the risky asset. It is not clear how this single parameter causes these very different results. It probably is related to the fact that when this noise is high, the precision of the forecast parameters is less crucial, so learning them precisely is less important. However, when this noise is low, these forecasts become more critical, and the system is very sensitive to how well the uninformed agents are learning their parameters.

This is a very interesting learning situation, and shows us just how subtle issues in learning can be. The question of how many agents are needed for good learning to occur in a population is an interesting one, and its relevance to real world situations may be important. This might be an issue which is only really addressable with computational frameworks, since many analytic methods are forced to use a continuum of agents, and the finite number problem goes away. The issue of parameter sensitivity will appear again in the next model.

2.5 The Santa Fe artificial stock market

The Santa Fe Stock Market is one of the most adventuresome artificial market projects. It is outlined in detail in Arthur, Holland, LeBaron, Palmer & Tayler (1997), and LeBaron, Arthur & Palmer (forthcoming 1998). This model tries to combine both a well defined economic structure in the market trading mechanisms, along with inductive learning using a classifier based system. This section gives a brief outline of the market structure along with a summary of some of the results.

The market setup is simple and again borrows much from existing work such as Bray (1982), and Grossman & Stiglitz (1980). In this framework, one period, myopic, constant absolute risk aversion utility, CARA, agents must decide on their desired asset composition between a risk free bond, and a risky stock paying a stochastic dividend. The bond is in infinite supply and pays a constant interest rate, r. The dividend follows a well defined stochastic process

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \epsilon_t, \tag{27}$$

where ϵ_t is gaussian, independent, and identically distributed, and $\rho = 0.95$ for all experiments. It is well known that under CARA utility, and gaussian distributions for dividends and prices, the demand for holding shares of the risky asset by agent i, is given by,

$$s_{t,i} = \frac{E_{t,i}(p_{t+1} + d_{t+1}) - p_t(1+r)}{\gamma \sigma^2_{t,i,p+d}}, \tag{28}$$

where p_t is the price of the risky asset at t, $\sigma^2_{t,i,p+d}$ is the conditional variance of $p+d$ at time t, for agent i, γ is the coefficient of absolute risk aversion, and $E_{t,i}$ is the expectation for agent i at time t. [12] Assuming a fixed number of agents, N, and a number of shares equal to the number of agents gives,

$$N = \sum_{i=1}^{N} s_i \tag{29}$$

which closes the model.

In this market there is a well defined linear homogeneous rational expectations equilibrium (REE) in which all traders agree on the model for forecasting future dividends, and the relation between prices and the dividend fundamental. An example of this would be

$$p_t = b + a d_t. \tag{30}$$

The parameters a and b can be easily derived from the underlying parameters of the model by simply substituting the pricing function back into the demand function, and setting it equal to 1, which is an identity and must hold for all d_t.

At this point, this is still a very simple economic framework with nothing particularly new or interesting. Where this breaks from tradition is in the formation of expectations. Agents' individual expectations are formed using a classifier system which tries to determine the relevant state of the economy, and this in turn leads to a price and dividend forecast which will go into the demand function.

The classifier is a modification of Holland's condition-action classifier, (Holland 1975, Holland, Holyoak, Nisbett & Thagard 1986), which is called a condition-forecast classifier. It maps current state information into a conditional forecast of future price and dividend. Current market information is summarized by a

[12] $E_{t,i}$ is not the true expectation of agent i at time t. This would depend on bringing to bear all appropriate conditioning information in the market which would include beliefs and holdings of all other agents. Here, it will refer to a simplified price and dividend forecasting process used by the agents. This demand function is valid if the shocks around the above expectations are gaussian. This is true in the rational expectations equilibrium, but it may not hold in many situations. We are assuming that disturbances are not far enough from gaussian to alter this demand. If they are, it does not invalidate the analysis, but it does break the link between this demand function and 1 period CARA utility.

bitstring, and each agent possesses a set of classifier rules which are made up of strings of the symbols, 1, 0, and #. 1 and 0 must match up with a corresponding 1 or 0 in the current state vector, and # represents a wild card which matches anything. For example, the rule 00#11 would match either the string 00111, or 00011. An all # rule would match anything. In standard classifier systems there is a determination made on which is the strongest rule depending on past performance, and the rule then recommends an action. Here, each rule maps into a real vector of forecast parameters, $a_{i,j}, b_{i,j}, \sigma^2_{i,j}$ which the agent uses to build a conditional linear forecast as follows,

$$E_{t,i,j}(p_{t+1} + d_{t+1}) = a_{i,j}(p_t + d_t) + b_{i,j}. \tag{31}$$

This expectation along with the variance estimate, $\sigma_{i,j}$ allows the agent to generate a demand function for shares using equation 28.

This is not the only way to build forecasts, and agents could be constructed using many other parametric classes of rules and forecasts. However, it does offer some useful features. First, the REE is embedded in the forecasts since the equilibrium forecast is linear in $p + d$. Second, if we force agents to decide on rules, using all information except for p_t in deciding which rule to use, the forecast gives a linear demand function in p_t above, which allows the market clearing price to be easily calculated.

The following is a list of the bits used to build conditional forecasts.

1-7 Price*interest/dividend $> 1/2, 3/4, 7/8, 1, 9/8, 5/4, 3/2$

8 Price $>$ 5-period MA

9 Price $>$ 10-period MA

10 Price $>$ 100-period MA

11 Price $>$ 500-period MA

12 always on

13 always off

These will be one when their conditions are true, and zero otherwise. Rules can be dependent on these information states, or they can ignore them. There are two potential problems in this for endogenous information usage. First, the agents are not able to use information outside of this restricted set. Second, the set itself may act as a focal point, increasing the chances that agents might coordinate on certain bits. In the critical rational expectations benchmark, all of these information bits should provide no additional information above and beyond the current price and dividend.

All matched rules are evaluated according their accuracy in predicting price and dividends. Each rule keeps a record of its squared forecast error according to,

$$\sigma^2_{t,i,j} = \beta\sigma^2_{t-1,i,j} + (1-\beta)((p_{t+1} + d_{t+1}) - E_{t,i,j}(p_{t+1} + d_{t+1}))^2 \tag{32}$$

This estimate is used both for share demand, and to determine the strength of the forecast rules in evolution. The GA strives to evolve rules with the lowest squared error. Obviously, the value of β is crucial in the above formula. It effectively determines the time horizon that agents are looking at in evaluating their rules.[13] If β is relatively small then the agents believe they live in a quickly changing environment. However, when it is large, they believe that the world they live in is relatively stable. In market simulations, β is set to an intermediate value and not changed. It would be nice in the future to allow this crucial parameter change over time.

The final important part involves the evolution of new rules. Agents are chosen at random on average every K periods to update their current forecasting rule sets. The worst performing 15 percent of the rules are dropped out of an agent's rule set, and are replaced by new rules. New rules are generated using a genetic algorithm with uniform crossover and mutation. For the bitstring part of the rules, crossover chooses two fit rules as parents, and takes bits from each parent's rule string at random.[14] Mutation involves changing the individual bits at random. Crossing over the real components of the rules is not a commonly performed procedure, and it is done using three different methods chosen at random. First, both parameters, a and b, are taken from one parent. Second, they are each chosen randomly to come from one of the parents. Third, a weighted average is chosen based on strength.

The results are performed with varying values of K. It is set either to very short horizon, frequent learning, at $K = 250$, or slower learning updates at $K = 1000$. In the case of slower learning, the resulting market behavior appears very close to the rational expectations benchmark. The agents learn to ignore the superfluous information bits, and the price time series is close to what it should be in the REE. In the actual time series from the market the information bits provide no additional forecasting power. A very different picture is revealed when the frequency of learning updates, K, is reduced to 250. At this point, the agents begin to use technical trading bits, and bits connected to dividend/price ratios. Also, the price time series support the fact that these pieces of information are indeed useful to agents. Finally, prices reveal volatility

[13]There are strong similarities between this and the horizon lengths in Lettau (1997).

[14]Selection is by tournament selection, which means that for every rule that is needed two are picked at random, and the strongest is taken. This type of crossover is referred to as uniform crossover.

persistence and increased trading volume relative to the slow learning case. All of these are features which have been observed in actual markets.

This is one of the most complex artificial markets in existence which brings both advantages and disadvantages. One thing this market does is to allow agents to explore a fairly wide range of possible forecasting rules. They have flexibility in using and ignoring different pieces of information. The interactions that cause trend following rules to persist are endogenous, they are not forced to be in the market. On the other hand, the market is relatively difficult to track in terms of a computer study. It is sometimes difficult to pin down causalities acting inside the market. This makes it harder to make strong theoretical conclusions about the reflections of this market on real markets.

2.6 Neural network based agents

An interesting market structure is put forth in Beltratti & Margarita (1992) and Beltratti, Margarita & Terna (1996). The setup is different from the earlier markets in that trade takes place in a random matching environment, and agents forecast future prices using an artificial neural network, ANN.[15]

Agents build a price forecast at time t, $E_t p_{t+1,j}$ using a network trained with several inputs including several lagged prices, and trade prices averaged over all agents from earlier periods. Agents are randomly matched and trade occurs when agent pairs have different expected future prices. They then split the difference, and trade at the price in between their two expected values. This relatively simple framework differs from some of the previous methods in that trade is decentralized. The impact this has on price dynamics and learning alone is an interesting question.

A further important distinction of this work is the addition of neural networks for modeling the agent forecasts. These are essentially nonlinear models capable of fitting a wide range of forecast structures.[16] None of the previously mentioned papers have employed these tools, and it interesting to see what they add to the learning dynamics.

Their most important results are concerned with looking at populations of different types of traders in which they allow the complexity, or intelligence, of traders to vary by changing the number of hidden units in the networks. This is essentially making the functions that the traders use for forecasting more or less complicated. Traders are allowed to buy a more complicated neural network at a given cost.[17] Actually,

[15] Another computational study looking at trade in a dispersed framework is Epstein & Axtell (1997).

[16] A useful neural network summary directed at economists is Kuan & White (1994).

[17] This is close in spirit to the heterogeneous information models such as Grossman & Stiglitz (1980), but now agents are purchasing forecasting complexity rather than actual information signals about a stock.

the neural nets are split into "smart" and "naive" with the smart having more hidden units and being more costly to use. They analyze stock price dynamics and the fraction of agents for each type. They find what appear to be cost levels at which both types can coexist, and other cost levels for which "smart" and "naive" dominate the market. Also, it appears that in the early stages of the market when prices are very volatile it pays to be "smart". Often after these initial periods have gone by the value added of extra forecasting power is reduced below its costs, and the "smart" agents disappear. This is an interesting result coming from this computational framework.

3 Other related studies

Unfortunately, covering all of the recent computational market models in great detail would be impossible. However, many other versions of artificial markets exist and these should be mentioned. For example, Rieck (1994) uses actual trading rules as in an evolutionary setting finding that they can have very interesting dynamics as the market evolves. Several other markets which appear to generate relatively realistic price dynamics include de la Maza & Yuret (1995), Marengo & Tordjman (1995), and Steiglitz, Honig & Cohen (1996). The intellectual history of many of these papers comes out of an area of research that postulates certain types of traders and watches what happens to market dynamics when they are put together. There are some connections to early adaptive expectations models, but most of this work does stress heterogeneity in the agent population.[18] Other approaches have brought in more sophisticated agents, as in De Long, Schleifer & Summers (1991), which also introduced the concept of noise trader risk. This paper also begins to address the question of whether a set of irrational traders might survive in the market, and finds that the answer to this question can be yes. Blume & Easley (1990) also address the question of whether rationality alone is a sufficient condition for survival in a financial market, and find this is not the case. There are strong connections between these basic questions about evolution applied to financial settings, and the computational models already considered.[19]

More sophisticated models move toward fully endogenizing the decisions about which trading mechanisms should be followed, and add social interactions across traders. Papers such as Brock & Hommes (1998), and Brock & LeBaron (1996) utilize discrete choice mechanisms and measures of past performance to model the decision making process of individual agents deciding whether to purchase a costly information signal, or

[18] Exampes include Bak, Paczuski & Shubik (1997), Cabrales & Hoshi (1996), Chiarella (1992), Day & Huang (1990), De Grauwe, Dewachter & Embrechts (1993), Frankel & Froot (1988), Gennotte & Leland (1990), Goldman & Beja (1979), Goodhart (1988), Levy, Levy & Solomon (1994), Savit, Manuca & Riolo (1997), and Zeeman (1974).

[19] There are also deeper questions about evolution in many areas of economics. See Hodgson (1993) for some examples.

use more sophisticated, but costly, forecasting models. Such models can also lead to interesting dynamics similar to the computational examples. In many cases in the equilibrium a zero cost, and very simple, forecast becomes optimal. This might be something like "the price follows a random walk." As all traders move toward using this forecast, the market no longer has the forecasting capabilities it would need to adjust to movements out of equilibrium. This makes it unstable, as price swings will not be damped out, and the agents will need to go back and purchase the costly forecasts.[20]

Another paper which is related to financial markets, but is not a financial model is Youssefmir & Huberman (1997). They model a resource allocation problem where forecasting what others agents are doing is crucial to decision making. Agents chose resources based on forecasts of how congested they will be. They can choose from a set of available forecasts, and use those that have performed well recently. Their interesting result is that the resource market shows bursts of volatility in resource allocations. This is similar to price volatility clumping in asset markets. The model is simple enough that the authors are able to provide an explanation for what is going on. For each "equilibrium" in the model there may be many forecasting rules that are consistent with it. As the system starts to settle into an equilibrium, agents move around randomly over the set of compatible rules. This random motion in the agent behavior can be strong enough to take the system back out of equilibrium in certain cases, and volatility starts up again. The authors give some simulations with large numbers of agents, and demonstrate that they are robust to many different types of model configurations. Just how generic a feature this is to populations of learning adapting agents in other situations is an interesting question. Further explorations will be needed to see if this is related to similar phenomenon in financial markets.

4 Designing Artificial Markets of the Future

At first it often seems like this area is a jumbled picture of many different models going in different directions. There is some truth to this in that the platforms and structures are not common, and broad comparisons across models are difficult. However, the contributions of these early papers are important in both showing what is possible using agent based methods, and laying a foundation for future work. They show that certain features of financial data which remain puzzling to single agent models may not be hard to replicate in the multiagent world. Also, they open up new theoretical questions about learning and information which are not present in more traditional models.

[20] Other papers that deal with simple agent populations that move around based on forecast performance are Kirman (1993), Linn & Tay (1998), Lux (1998), Topol (1991), and Sacco (1992).

Even though these papers are fundamental to this area, it would be premature to lock in the design choices that they have made. The computational realm has the advantages and disadvantages of a wide open space in which to design traders, and new researchers should be aware of the daunting design questions that they will face. Most of these questions still remain relatively unexplored at this time. Several of the crucial ones will be summarized here. Unfortunately, there are no easy answers to many of these questions.

An obvious choice that one makes is in agent design and data representation. The models discussed here represent a big range of possible agent constructions, but this is far from complete. It is important to realize that many results may be heavily influenced by the learning methods decided on for agents. Even relatively free form methods such as artificial neural networks have an impact on the types of behavior that the learning agents are capable of. One example of this from the SFI market is whether the classifiers using technical trading bits as information caused these to be endogenously chosen by the agents. Also, most all of the learning methods considered put explicit bounds on the level of complexity possible for agents. This ad hoc limitation makes the computerized models not all that different from some of the explicit trading rule models, and leaves open the question "Could a really smart trader have made money, and changed the patterns in the market?"[21] Methods such as genetic programming which are freer of structural constraints might be useful, but will be more difficult to analyze. The best models for expost analysis are the simpler ones where policy functions are constructed and evolved over time, but these limit the types of learning dynamics that can be studied.

Outside of the very practical question of agent representation are some deeper questions about learning and evolution. Financial markets often seem like an easy place to set up well defined learning and fitness objectives. However, it isn't that simple. The objective of utility maximization may not line up with a natural evolutionary objective of long run wealth maximization.[22] This causes some interesting questions for the choice of individual objective functions. Should one stay close to the individual maximization frameworks we are used to even though these agents might be far from maximizing their survival rates, and the modeler might be spending a lot of time constructing agents that will disappear in the long run. Many of these questions are also directly related to the issue of time horizons. If agents are trying to decide between behavioral rules, what horizon should they use to evaluate them? They must try to build some estimate of the future performance of a trading method, but how far into the past should they test it? Is it likely that things have changed, and only short histories are relevant? This may be one of the critical points

[21] A nonfinance example of allowing representations to adjust complexity is given in Lindgren (1992).

[22] See Blume & Easley (1990) for examples.

appearing in the SFI market. Time horizons and stationarity are important. Possibly, even more important than details of the objective functions. Unfortunately, little guidance is available on what to do about this interesting question.

Another deep agent question that affects all agent based models is the question of learning versus evolution. Are these models of agents learning over time, or should they be thought of as new agents entering the market according to some birth and death process? This question is definitely one that is considered by others in the learning and evolution literature. The models described in this survey do not approach this in the same fashion. This is not a problem, but it should be realized that different dynamics are at work. Learning, but infinitely lived, agents may be more likely to come up with robust behaviors able to adapt to many situations, whereas finite lived agents might be specific to the current environment. Which is better for representing financial markets is not clear.[23] Another related subject is how much testing of strategies may go on without actually using them. An agent might maintain a set of rules, many of which are simply being tested off line, but not actually traded. In other words parts of the population might simply be observing, but not actively participating in the market.[24] If agents believe they cannot affect prices then it would be sensible for them to monitor other rules, or other agents' trading behaviors. How much of this goes on is not clear, but this is another dimension on which economic systems differ from their ecological counterparts.[25]

Aside from these issues, very simple practical and important questions remain. Among these are the question of what type of trading mechanism should be used. For markets that might be out of equilibrium this important decision may have major consequences. It may be important to study the dynamics of different trading setups for markets in the hope of gaining insight into crucial policy questions about their dynamics. However, researchers looking for more general properties of learning and financial markets may be frustrated by the fact that results are sensitive to the actual details of trading. Artificial markets may be based on actual market mechanisms such as the double auction, or they may be based on temporary price equilibrium such as the SFI market. The need for a centralized market is also an interesting question. Of the papers considered here only Beltratti et al. (1996) used a noncentralized trading mechanism. As learning mechanisms are better understood there will also be a parallel need to understand the impact of trading mechanisms. Of all the issues mentioned here, these may have the most direct policy relevance.

Validation remains a critical issue if artificial financial markets are going to prove successful in helping to

[23] There is a strong overlap between this question and the issue of time horizons mentioned previously.

[24] There are some connections from this to the contrast between individual and social learning studied in Vriend (1998).

[25] This has a weak connection to the famous exploitation versus exploration tradeoff. This describes the decisions one needs to make in testing out new strategies. The strategies provide both an expected payoff to an agent, and information about about its performance. Given that agents are small and do not affect prices in financial settings, this may not be a big problem in that many rules can be explored without actually implementing them.

explain the dynamics of real markets. This remains a very weak area for the class of models described here. Further calibration techniques and tighter tests will be necessary. These issues affect all types of economic models, and in many ways the basic problems are not different. However, there are some key issues which affect these markets in particular. First, they are loaded with parameters which might be utilized to fit any feature that is desired in actual data. Second, they can be exceedingly complicated, almost to the point of losing tractability. Obviously, a move toward keeping the models relatively simple without cutting out their key components is necessary, but it is difficult to decide along which dimensions things should be cut out.[26] Artificial markets can try to battle the proliferation of parameters by trying to set the empirical hurdles very high. Examples of this might be to line up with many different data sets, and different time horizons. Also, the use of experimental data, and the potential design of experiments to falsify certain computer models remains a largely unexplored area.

5 Conclusions

It would be a long stretch to say that artificial financial markets have solved many of the empirical puzzles that we face today in financial markets, but they provide new and very different approaches to traditional economic modeling. Viewing markets as very large aggregations of agents with heterogeneous beliefs and goals often reveals very different perspectives on traditional theoretical thinking. To hold up as serious theoretical structures for researchers and policy makers it is clear that more work on validation and testing is needed. This is not a trivial problem in this area since the computer markets are often heavily parameterized and very complex. However, most of the markets considered here are very simple in the dimension of actual agent goals and objectives. It is their methods for forecasting the environment around them, and the interactions of these forecasts that become complex. In many ways these simpler agents are closer to the goal of explaining economic phenomenon with highly stylized and simple representations of the underlying decision making process.

[26] The computer modeler faces a different set of problems from the analytic modeler here. Often the latter is faced with constraints about what can be done analytically. This pushes to keep the framework simple, but the simplicity is due to analytic tractability rather than economic structure. The computer modeler is free of these constraints, but this can be both a blessing and a curse, in that it can lead to overly complicated structures which are difficult to examine.

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