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An Explanation of Generic Behavior in an Evolving Financial Market

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Abstract

The Santa Fe Artificial Stock Market [13, 4] is an agent-based artificial model in which agents continually explore and develop expectational models, buy and sell assets based on the predictions of those models that perform best, and confirm or discard these models based on their performance over time. The purpose of this paper is to classify the different types of behavior that emerge in the market as a function of evolutionary learning rate, and to explain these emergent behaviors. We observe four different types of behavior, which are distinguished by their effects on the volatility of prices, the complexity of strategies, and the wealth earned by agents over time. We also show that the differences between these behaviors may be attributed to variations in the rate at which agents revise their trading rules and the subsequent types of rules—technical or fundamental—that emerge in the market.

1 Introduction

Financial markets are complex. Their booms and crashes [15, 16, 17], distinct moods [1], and non-linearities [14, 8, 9] all blunt the analytical tools of traditional economic theory. Reexamination of financial market behavior with the new techniques of agent-based economic modeling is now suggesting that this type of complexity may be an intrinsic property of such systems [13, 4, 10, 7].

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The Santa Fe Artificial Stock Market, developed by Brian Arthur, John Holland, Blake LeBaron, Richard Palmer, and Paul Taylor at the Santa Fe Institute, provides a compelling example of how simple endogenous forces can cause complex market behavior. Arthur et al. [13, 4] showed that varying the rate at which individual agents learn new investment strategies reveals two different kinds of overall market behavior. If investment strategies evolve slowly, the market showed behavior generally consistent with the prediction of traditional economic theory. But if the strategies were allowed to evolve more quickly, the market showed the kind of instabilities and statistical properties typically observed in real-world markets. Their work suggests that the cause of the complex behavior of financial markets may involve the rate at which investment strategies evolve.

This paper follows up on the work of Arthur et al. by taking a closer look at the kinds of behavior exhibited by the Santa Fe Stock Market model. We systematically study how the market's behavior depends on the rate of evolutionary learning, classify the various behaviors that emerge, and attempt to explain these behaviors. The main novelty of the present study is the light shed on market behavior by the historical patterns in the activation of investment strategies.

2 The Santa Fe Artificial Stock Market

The artificial stock market we study here was developed by Brian Arthur, John Holland, Blake LeBaron, Richard Palmer, and Paul Taylor [13, 4]. The market consists of a population of heterogeneous agents that buy, sell, and hold stocks and bonds. An agent's buy, sell, and hold decisions are made on the basis of that agent's beliefs about whether the stock's dividend is likely to go up or down, and those beliefs are determined by a set of market forecasting rules that are continually being assessed as to accuracy. Over time an agent's set of market forecasting rules evolve under the action of a genetic algorithm.

The following sections provide a brief introduction to the Santa Fe Artificial Stock Market model. More detailed descriptions are available elsewhere [13, 4]. When mentioning some of the model parameters below, we indicate the specific parameter values we used in the work reported here with typewriter font inside brackets [`like this`].

2.1 The Market

The market contains a fixed number N [`25`] of agents that are each initially endowed with a certain sum of money (in arbitrary units) [`1000`]. Time is discrete. Each time period each agent must decide whether to invest her money in a risky stock or in a risk-free asset analogous to a real world Treasury Bill. The risk-free asset is in infinite supply and pays a constant interest rate r [`10%`]. The risky stock, issued in N shares, pays a stochastic dividend that varies over time. The stock's dividend stream is an exogenous stochastic process whose present value is unknown to the agents.

Agents apply their market forecasting rules to their knowledge of the stock's price and dividend history to perform a risk aversion calculation and decide how to invest their money at each time period. The price of the stock rises if the demand for it exceeds the supply, and falls if the supply exceeds the demand. Each agent in the market can submit either a bid to buy shares, or an offer to sell shares—both at the current price p_t—or neither. Bids and offers need not be integers; the stock is perfectly divisible. The aggregate demand for the stock cannot exceed the number of shares in the market. The agents submit their decisions and offers to the market specialist—an extra agent in the market who controls the price so that his inventory stays within certain bounds. The specialist announces an initial trial price, collects bids and offers from agents at that price, from these data announces a new trial price, and repeats this process until demand and supply are equated. The market clearing price serves as the next period's market price.

2.2 Agents and Market Forecasting Rules

Agents possess a constant absolute risk-aversion utility function of the form $U(c) = -exp(-\lambda c)$, where λ [`0.5`] measures the extent of risk aversion and $0 < \lambda \leq 1000$. At each time period each agent determines the number of shares and risk-free bonds that maximizes her utility of consumption. The outcome of this decision depends on the agent's estimate of the profitability of the stock and bond.

The agents make their investment decisions by using a set of hypotheses or rules about how to forecast the market's behavior. At each time period, each agent considers a fixed number [`100`] of forecasting rules. The rules determine the values of the variables a and b which are used to make a linear forecast of next period's price:

$$E(p_{t+1} + d_{t+1}) = a(p_t + d_t) + b$$

where p_t is the trial price and a and b are the forecasting parameters. The forecasting rules have the following form:

IF (the market meets condition D_i) THEN $(a = k_j, b = k_l)$

where D_i is a description of the state of the market and k_j and k_l are constants.

Market descriptors (D_i) match certain states of the market by an analysis of the price and dividend history. The descriptors have the form of a boolean function of some number [`12`] of market conditions. The set of market conditions in each rule is represented as an array of bits in which `1` signals the presence of a certain condition, `0` indicates its absence, and `#` indicates "don't care". The breadth and generality of the market states that a rule will recognize is proportional to the number of `#` symbols in its market descriptor; rules with descriptors with more `0`s and `1`s recognize more narrow and specific market states. As these strings are modified by the GA, the number of `0`s and `1`s might go up, allowing them to respond to more specific market conditions. An appropriate reflection of the complexity of the population of forecasting rules possessed by all the agents is the number of specific market states that the rules can distinguish, and this is measured by the number of bits that are set in the rules' market descriptors.

There are two different kinds of market conditions: those pertaining to trends in the stock price, which are recognized by *technical* trading bits, and those pertaining to the relationship between the stock's price and its fundamental value, which are recognized by *fundamental* trading bits. So, there are two (overlapping) kinds of rules, depending on whether their descriptors have technical or fundamental bits set. Technical trading rules are activated when the current state of the market meets some condition pertaining to a price trend (e.g., the condition that the current stock price exceeds the average price over the past fifty time periods). Fundamental trading rules are activated when the current state of the market meets a condition pertaining to the relation between the stock's price and fundamental value (e.g., the condition that the the current stock price times the interest rate divided by the most recent stock dividend exceeds 0.75). This method of modeling expectation formation makes it is possible to track exactly which descriptor bits (technical or fundamental) are being used by agents in the model, and this allows us to study the conditions under which technical trading emerges in the market.

An example may help clarify the structure of market forecasting rules. Suppose that there is a twelve bit market descriptor, the first bit of which corresponds to the market condition in which the price has gone up over the last fifty periods, and the second bit of which corresponds to the market condition in which the price was 75% higher than its fundamental value. Then the descriptor `10##########` matches any market state in which the stock price has gone up for the past fifty periods and the stock price is not 75% higher than its fundamental value. The full decision rule

IF `10##########` THEN $(a = 0.96, b = 0)$

can be interpreted as "If the stock's price has risen for the past fifty periods and is now not 75% higher than its fundamental value, then the (price + dividend) forecast for the next period is 96% of the current period's price."

If the market state in a given period matches the descriptor of a forecasting rule, the rule is said to be *activated*. A number of an agent's forecasting rules may be activated at a given time, thus giving the agent many possible forecasts to choose from among. The agent decides which of the active forecasts to use by measuring each rule's accuracy and then choosing at random from among the active forecasts with a probability proportional to accuracy. Once the agent has chosen a specific rule to use, the rule's a and b values determine the agent's investment decision.

2.3 The Genetic Algorithm

A genetic algorithm (GA) provides for the evolution of the population of forecasting rules over time. Whenever the GA is invoked, it substitutes new forecasting rules for a certain fraction [`5%`] of the least fit forecasting rules in each agent's pool of rules. A rule's fitness is determined by both how well it has performed and by how complex it is (the GA has a bias against complex rules). Applying the genetic operators of mutation, crossover, and inversion to the most successful rules in the agent's rule pool creates the new rules, with more accurate rules producing more offspring. New rules are assigned an initial accuracy by averaging the accuracy of their parent rules.

The only market parameter that we varied in the results described below is the waiting time between invocations of the GA. We term this waiting time between GA invocations the *GA interval*. So, if the GA is invoked every time period, GA interval is 0; if the GA is invoked every 1000 time periods, GA interval is 1000; if the GA is never invoked, GA interval is 300000 (this was the total length of the simulation).

The model contains another mechanism for changing an agent's rules. If some agent's rule is not activated (thus not considered for use) by an agent for a significant number of time periods [`1000`], then one of the bits in the rule that is set is changed to a `#` so that it matches a broader set of market states. This makes it more likely to be activated and used by agents in the market.

3 Experimental Methods

We systematically studied how the behavior of the market depends on a key model parameter identified in earlier work—the interval between successive invocations of the genetic algorithm (GA), which we will term the "GA interval." Previous experiments with GA interval [13, 4, 12] simultaneously varied the probability of crossover and the accuracy updating parameter.[1] Here, we fix the crossover probability at 0.3 and the accuracy updating parameter at .01. All simulations were run for $300,000$ time periods in order to make the results independent of the initial random assignment of forecasting rules and to allow the asymptotic properties to emerge. We collected statistics on stock prices, stock trading volumes, accumulated wealth of agents, and number of bits set (technical and fundamental) in forecasting rules.

In order to explain the behavior we observed, we also collected data on the activation histories of various rules during a simulation. The activation history at time period t is the number of times a particular rule has been activated until time period t (summed over time). If a hypothesis is activated but not used, in one way or another it will eventually be removed by the genetic algorithm. So a rule's activation history us a rough indication of the number of times it has actually been used by an agent in the market.

[1] Unpublished results involving variation in GA interval alone have been mentioned in a footnote in [4].

4 Results

We observed four distinct types of behavior in the model, corresponding to four kinds of evolutionary learning. Two have been previously noted [13, 4]; the other two are boundary conditions. The differences between the four kinds of behavior can be seen in the volatility of prices, the wealth earned by agents (Figure 1), the total number of bits that are set in the forecasting rules, the relative number of technical and fundamental bits set (Figure 2), and the activation histories of the rules used by agents (Figures 3 and 4). Other differences (not shown here) can be seen in the mean prices, the trading volumes, and the deviations of the stock price from its fundamental value. The four classes of behavior can be summarized as follows, starting with the two boundary conditions:

Class I: No evolution so no rule switching. When the GA is never invoked (GA interval is the length of the simulation, i.e. $300,000$ time periods), the agents have no choice but to stick with the pool of hypotheses with which they were initially endowed. The main characteristics of this regime are low volatility of prices, low accumulated wealth, and similar levels of fundamental and technical trading.

Class II: Too fast evolution prevents rule switching. When the GA is invoked at every time period (GA interval is 0), the prices are very stable, the complexity of strategies is very low, there is no significant difference between technical and fundamental trading, and wealth earned is high.

Class III: Slow evolution enables only slow rule switching. When the GA interval is moderately low ($1000 \leq$ interval ≤ 10000), price volatility is moderately low, the complexity of forecasting rules is low, wealth earned is high, and technical trading is low. In previous work the model authors noted that this class of behavior is consistent with the predictions of the theory of Rational Expectations and the efficient markets hypothesis is finance, so they called this the Rational Expectations (RE) regime [13, 4].

Class IV: Fast evolution encourages frequent rule switching. When the GA interval is moderately high ($100 <$ interval ≤ 1000), prices are volatile, the complexity of strategies is very high, wealth earned is low, and there is significant technical trading. The model authors observed that prices in this class of behavior deviate significantly from their fundamental values, bubbles and crashes occur frequently and the market shows statistical properties similar to real world stock markets [13, 4] They called class IV the Complex Regime.

Classes I and II are very similar but we classify them separately because their behavior has significantly different causes. In Class II the GA is invoked at each time step and so the pool of decision rules is constantly changing, whereas in Class I the GA is never invoked and the pool of rules undergoes no changes at all. The behavior seen in Class II arises from a market that appears to be somewhat chaotic, even though it resembles a regime that is the exact opposite.

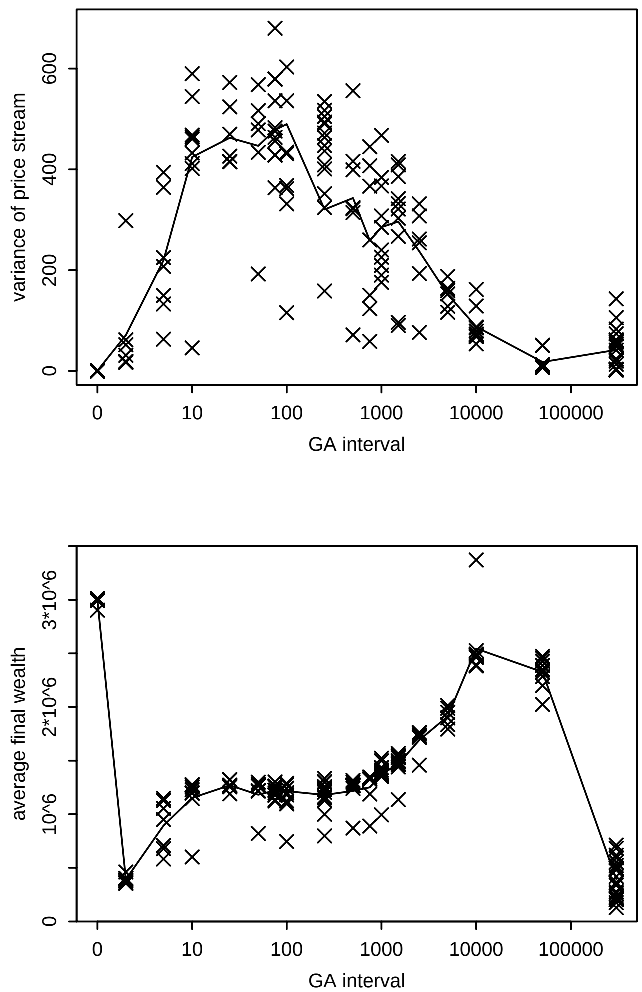

Figure 1: Above: Variance of the stock price time series as a function of GA interval. A line showing the mean variance values at each GA interval overlays a scatter plot of variance values from all the simulations at various GA intervals. The far left of the GA interval scale represents interval zero. Note that variance is very low at the two boundary conditions (very small and very large intervals), and that between those boundary conditions variance is proportional to GA intervals. Below: Average final wealth of investors in the market as a function of GA interval. A line showing the average final wealth values at each GA interval overlays a scatter plot of wealth values from all the simulations at various GA intervals. Comparison with variance of the price stream (above) shows that investor accumulated wealth is inversely proportional to variance of the stock price stream between the two boundary conditions.

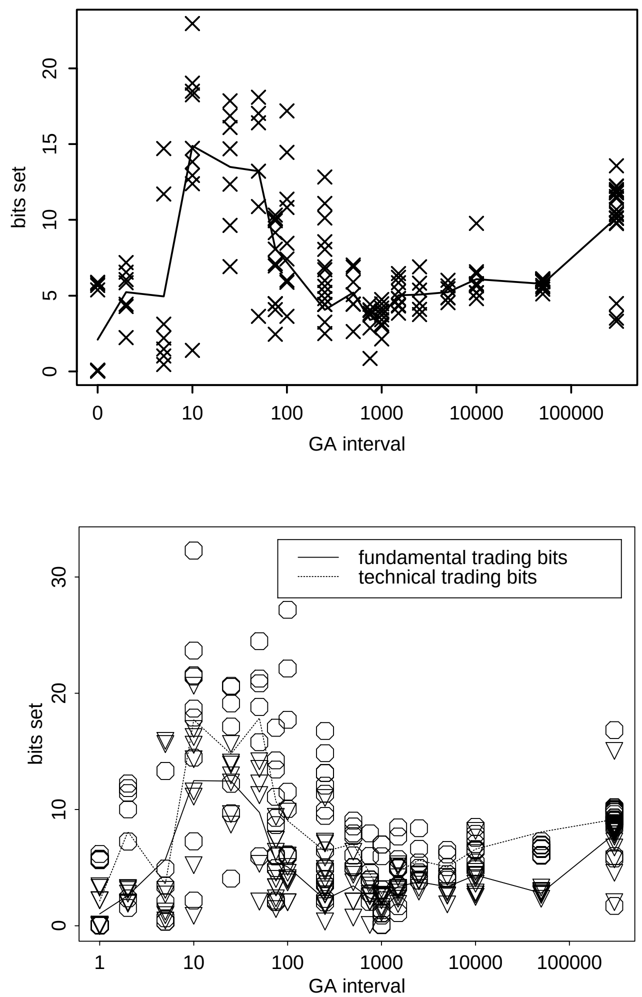

Figure 2: Number of bits in each agent's pool of trading strategies that are set to non-null values (a measure of strategy complexity) as a function of the GA interval. A line showing the average number of bits set at each GA interval overlays a scatter plot of data from all the simulations. Above: all bits are graphed together. Below: technical trading bits (open triangles) and fundamental trading bits (open dots) are graphed separately. The number of bits set is normalized (i.e., divided) by the total number of bits available. The number of bits set at very large GA intervals simply reflects the number of bits set in the initial population of strategies; the GA cannot change the strategy bits if it virtually never runs. When the GA interval does significantly change the complexity the strategies, large interval GA *lowers* it, small interval GA *raises* it, and very small GA interval *lowers* it.

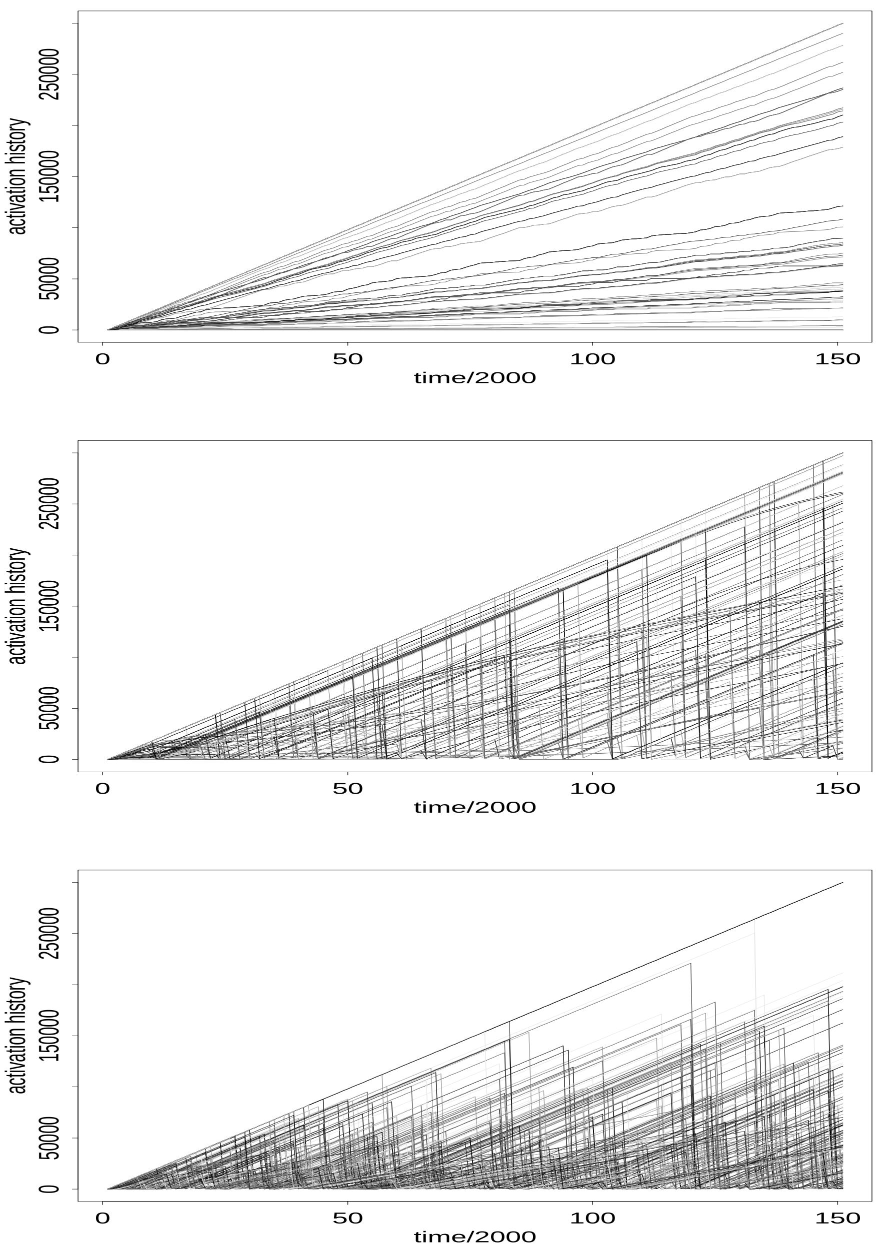

Figure 3: Activation of the investors' individual trading strategies as a function of time, at three GA intervals. Top: GA interval is 300,000; the GA never runs. Middle: GA interval is 10000; the GA runs 30 times in 300,000 time periods. Bottom: GA interval is 1500; the GA runs 200 times in 300,000 time periods.

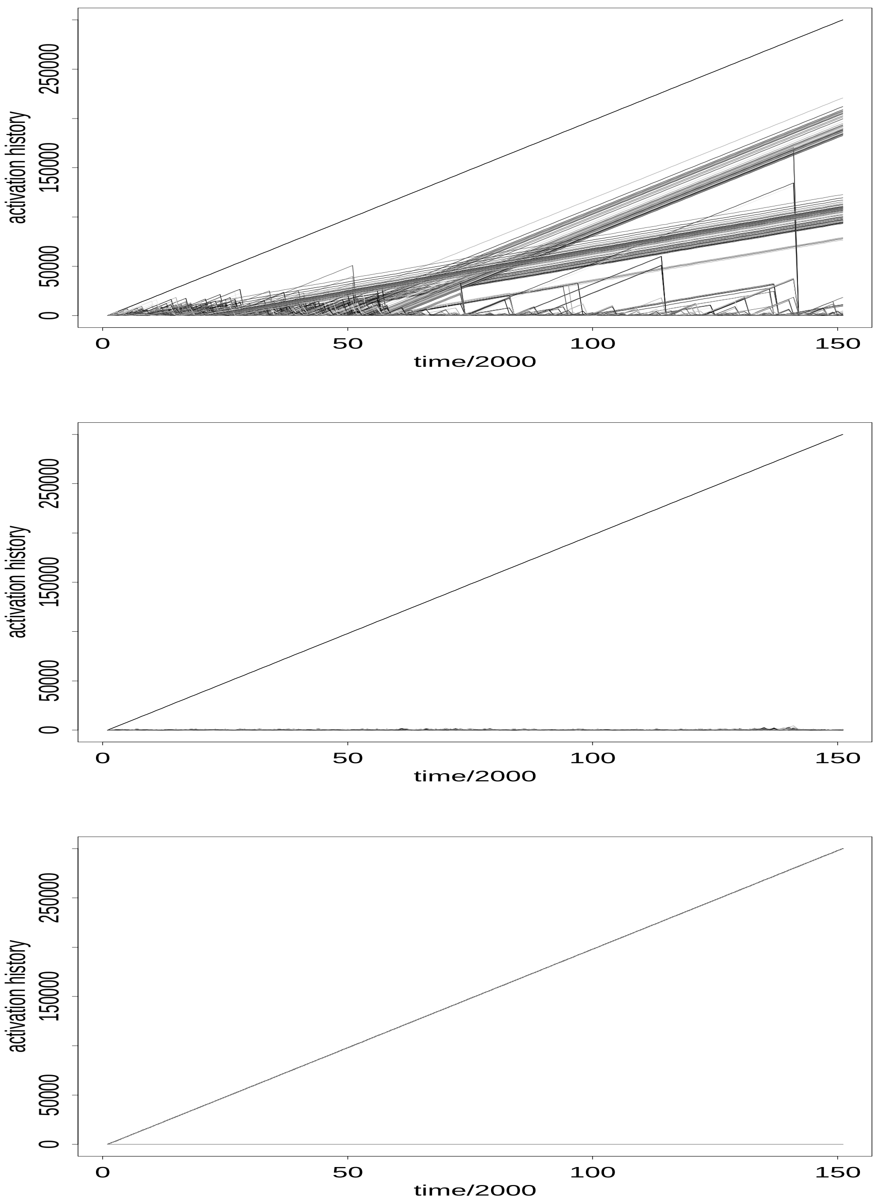

Figure 4: Activation of the investors' individual trading strategies as a function of time, at three GA densities. Top: GA interval is 250; the GA runs 1250 times in 300,000 time periods. Middle: GA interval is 5; the GA runs 60,000 times in 300,000 time periods. Bottom: GA interval is 0; the GA runs every time period.

It is important to note that the classes described above are separated by periods of transition. At GA interval of 5 for example, the market shows characteristics of Class II and Class III behavior. The time series data of stock prices, wealth, technical and fundamental trading and the complexity of strategies appear to belong to class III, and the underlying behavior resembles both Class II and Class III(Figure 4).[2]. An interesting topic future research is to investigate the exact nature of the transition between these classes.

5 Discussion

The four different classes of behavior described above may be attributed to the effects of GA invocation rates on agent's evolutionary's learning. Evolutionary learning affects the rate at which the agents switch between trading strategies. At the boundary conditions (GA interval 0 and GA interval 300,000) evolutionary learning is virtually nonexistent and so there is no significant evolution of trading strategies. Since the agents' trading strategies are relatively stable, so is the price series in the market. By contrast, when the GA interval is moderately low or moderately high, evolutionary learning is significant and this leads the agents' trading strategies to evolve, and this in turn makes the market less stable.

The speed at which agents switch strategies also affects the *type* of rules that they use: technical trading is significantly higher when the GA interval is moderately small. One explanation of this effect, developed below, depends on the connection between the "breathing time" a new rule enjoys before being scrutinized by the GA. Arthur et al. provide an additional explanation of this effect [4]: When GA interval is small, the agents switch rules often enough that it becomes likely for similar technical trading rules to be used by other agents in the population. Technical trading rules, when used by enough agents, can become self-fulfilling prophesies—if enough people believe the stock price is due to increase and buy the stock as a result, their demand for the stock will drive up the price—thus leading to market bubbles and crashes. Market volatility is roughly proportional to the presence of technical trading, so the regimes with less technical trading are significantly more stable.

In class I with GA interval at or near $300,000$, the same pool of market forecasts available to the agents virtually never changes. The number of technical and fundamental bits set in the population of forecasting rules (Figure 2) reflects the complexity of the rules randomly assigned at the start of the simulation. In addition, as Figure 3 (top) shows, the rate at which different forecasting rules are activated by the market states is quite constant over time, and presumably the rules the agents actually use is similarly constant. In fact, fully a quarter of all of the available rules are activated virtually every time period, and thus

[2]The activation history graph (Figure 4)shows that that the set of strategies used by agents is quite stable over time. This makes it similar to Class II. But unlike class I, some other strategies are also used (though not as frequently as the set of stable strategies). This makes it resemble Class III

contribute to the slope 1 line in the Figure. The agents' behavior becomes quite stable and predictable, which makes the market stable and predictable in turn, as Figure 1 (top) shows. (We are unsure why average final wealth in this regime varies as observed in the bottom of Figure 1.)

In class II with GA interval at or near 1, the GA's continual operation causes continual flux in the population of rules available to the agents. Yet, as Figure 3 (bottom) shows, virtually always the same subset of forecasting rules is activated. Furthermore, close to 95% of the available rules contribute to the slope 1 line representing these continually activated rules. The rules the agents actually use are chosen from these continually reactivated rules, of course, so Figure 3 (top) shows that the agents' trading strategies are stable over time. Thus, although there is a continual flux in the population of rules, the subset of rules actually used virtually never changes. The same subset (5%) of rules is continually replaced by the GA. Thus in class II the genetic algorithm only generates useless hypotheses so the rules being used never changes. As in class I, this stability of forecasting strategies makes the market relatively stable and predictable, as Figure 1 (top) shows. Figure 2 shows that class II evolution produces simpler strategies. This is probably due to the built-in cost of set bits, i.e., the evolutionary bias toward simpler strategies. If evolution cannot build useful strategies, as class II evolution evidently cannot, then simpler strategies should prevail. (We are unsure how to explain the variation in average final wealth seen in the bottom of Figure 1.)

Class III behavior appears when the GA interval is moderately large, roughly $1000 \leq \text{interval} \leq 10000$. The GA is invoked frequently enough for evolutionary learning to significantly improve the agents' strategies, unlike in the boundary conditions which cannot support evolutionary learning. The accumulated wealth in Figure 1 (bottom) shows the value of the strategies that evolutionary learning can produce. Only 4% of the rules are continuously activated—they are the rules that contribute to the slope 1 line in Figure 3 (bottom)—so the rules the agents actually use continue to evolve over the course of the simulation. The agents switch their investment strategies, but only relatively slowly. At the same time, the waiting time between GA invocations is long enough that newly generated rules have a relatively long time to prove their worth before they face selection pressure from the GA. This means that evolutionary learning has an opportunity to discover those forecasting rules that are successful only in the long run (technical trading rules that identify very long-term trends or fundamental trading rules that do well only over the long haul). To the extent that agents are using rules that are successful only over the long haul, their rule use will tend to be fairly stable over time. This explanation would predict the kind of rough correlation between GA interval and price stream variance visible in the class III portion of Figure 1 (top), and the agents' risk aversion explains class III's inverse correlation between price stream variance and average final wealth (Figure 1). Evidently, these rules that focus on the long-term are not especially complex, so the GA bias toward simpler rules probably explains the relatively low complexity of class III rules (Figure 2).

Class IV behavior happens when the GA interval is moderately small, roughly

$\times 1000 \leq$ interval ≤ 100. Figure 1 (bottom) shows that agents are able to accumulate some significant wealth, so the GA interval is not so low that it disables evolutionary learning. Yet the waiting time between GA invocations is short enough that rules must prove their worth relatively quickly to avoid succumbing to the GA. This sort of evolutionary learning favors rules that perform well in the short run. As with class III, only 4% of the rules are continuously activated; Figure 4 (top) shows that the subset of rules that the agents actually is continually evolving. Agents are switching their investment strategies relatively quickly. This instability in investment strategies used causes instability in the stock price (Figure 1 top), and the market becomes less predictable than in any other regime. Given the agents' risk aversion, this market instability drives the price down (Figure 1 bottom), Figure 2 shows not only that the rules produced in class IV are relatively complex and use more trading bits than those in any other class; the complexity of the quickly evolving trading strategies provides enough value to outweigh the GA's built-in bias toward simple rules. In class IV, and only in class IV, evolutionary learning supports the emergence of significantly complex strategies, and complex technical trading strategies in particular.

6 Summary and Conclusion

Varying the interval of the GA in the Santa Fe Stock market results in the appearance of four distinct kinds of market behavior. These correspond to four different rates of evolutionary learning. Evolutionary learning controls the rate at which agents switch between different rules in the population of rules. It also affects the types of different strategies (technical or fundamental) that evolve over time. Differences between rates of switching between rules and the types of rules that evolve in these classes lead to differences in the volatility of prices, wealth earned by agents, the complexity of strategies, the types of strategies that evolve in the market over time and the activation history of rules.

At low GA intervals, the frequent switching between strategies as well as the significant usage of technical trading rules results in high price volatility, increases in the complexity of strategies and lower overall wealth. At longer GA intervals, the infrequent switching between rules as well as the lower usage of technical trading rules results in lower price volatility, the usage of strategies of lower complexity and higher overall wealth. At the boundary conditions the usage of the same pool of rules over time leads to very low volatility and almost equal usage of technical and fundamental rules.

In conclusion, this paper has classified the various types of behavior in the Santa Fe Stock market and provided an explanation for the differences between observed behaviors. Given the resemblance of Class IV behavior to real world financial markets [4, 12], we hope that our results are also a step toward explaining the complexity of real world financial markets. Current and future work in this area includes quantifying evolutionary activity in this model using neutral models and evolutionary activity statistics [5, 6], and also studying the emergence of technical trading in financial markets[11].

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