Avalanche Dynamics and Trading Friction Effects on Stock Market Returns

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Abstract

We propose a model with heterogeneous interacting traders which can explain some of the stylized facts of stock market returns. A generalized version of the Random Field Ising Model (RFIM) is introduced to describe trading behavior. Imitation effects, which induce agents to trade, can generate avalanches in trading volume and large gaps in demand and supply. A trade friction is introduced which, by responding to price movements, creates a feedback mechanism on future trading and generates volatility clustering.

Keyword: Stock market models, volatility clustering, complex systems.

I. INTRODUCTION

A variety of market models based on the ideas of statistical physics have been introduced in the last few years (see [1] for a recent review). Some of these [2–4] address the problem of slowly decaying autocorrelation of absolute returns, a phenomenon known in the literature as volatility clustering [5–9].

The analysis of market indexes and exchange rates [10–13] shows that the volatility autocorrelation functions are power-laws over a large range of time lags, and multi-scaling behavior of volatility autocorrelations has also been detected [14–16]. This is in contrast with the outcome of the popular ARCH-GARCH models [6,17] which successfully describe the presence of long term correlations on return volatility but predict an exponential decay for it.

Econometricians have devoted considerable attention in the last years in detecting co-movements of volatility with other economic variables in the attempt to interpret and capture the source of clustering effects in returns. In particular a lot of work has been devoted to the analysis of interaction between volatility of returns and trading volume. Daily financial time series provide empirical evidence [18,19] of a positive autocorrelation, with slowly decaying tails, for the trading volume, and positive cross correlation between trading volume and returns volatility.

In this paper we will propose a mechanism which correlates price fluctuations and changes in trading volume based on ideas coming from the physics of disordered systems.

The market micro-structure literature is based on the perspective that price movements are caused primarily by the arrival of new information [20,21]. By contrast, in the physics community instead, and also among many economists, the idea that price fluctuations are due to the interaction among market players and the trading activity, is becoming widely accepted. For example herd behavior [22–26,2,4] has been proposed as a possible form of interaction to explain the observed statistical outcomes in financial markets.

Another important element which has been introduced in modeling the trading activity is the heterogeneous character of the agents. For example, if aggregate news could be symmetrically accessed and quickly transmitted, communication would be superfluous unless traders reacted differently to its arrival.

Physicists are used to approaches based on the idea of *universality*: relevant mechanisms are often independent of the details of individual behavior, and just depend on very general features of the problem. Applying this approach to the stock market, one can hope to understand some features of stock price fluctuations in terms of only the interrelation between traders and the statistical distribution of their characteristics.

II. THE MODEL

A modified version of the random field Ising model (RFIM) is employed to describe the behavior of agents in the stock market. Sethna and coworkers introduced the short range RFIM [27,28] to model the magnetization of ferromagnetic systems in the presence of an external driving field. In the limit of quasi-static driving the model is known to display a disorder-induced continuous transition between a regime of smooth hysteresis cycles, displaying only finite size avalanches, and a regime of hysteresis cycles with sharp magnetization jumps, corresponding to percolating avalanches. At the critical point, the model generates avalanches of all sizes and durations, whose statistical distributions take the form of power-laws. This disorder-induced criticality appears also in the random bond Ising model (RBIM), the RFIM, in a version of the RFIM with three possible spin states (RFBEG), and in the diluted RFIM. The effects of a finite rate driving on the hysteresis cycle and avalanches have also been numerically investigated [29].

The RFIM has previously been employed to model social behavior by Galam [30]. For earlier papers on Ising market models see [31,32]. As will become clear in the following, we will consider a modified version of the RFIM model allowing the spin variables to take three possible values: $+1$, -1 and 0, and introducing a dynamics for the local random fields, which depart from the usual quenched approximation.

We consider an $L \times L$ square lattice with periodic boundary conditions. Each node i is occupied by one agent.

We start with each agent initially owning the same amount of capital consisting of two assets: cash $M_i(0)$ and $N_i(0)$ units of a single stock. At any time step t the capital of trader i is given by $K_i(t) = M_i(t) + p(t)N_i(t)$, where $p(t)$ is the current price of the stock. At each time step t a given trader, i, chooses an action $S_i(t)$ which can take one of three values: $+1$ if (s)he buys one unit of the stock, -1 if (s)he sells one unit of the stock, or 0 if (s)he remains inactive. The trades undertaken by each player are bounded by his resources plus the constraint that (s)he can buy or sell only one indivisible unit at a time.

The agents' decision making is driven by noise and the influence of their nearest neighbors. At each time t, each agent i receives a signal $Y_i(t)$:

$$Y_i(t) = \sum_{<i,j>} J_{ij} S_j(t) + A\nu_i(t) + B\epsilon(t) \tag{1}$$

$< i, j >$ denotes that the sum is taken over the set of nearest neighbors of agent i. J_{ij} measures the influence exercised on agent i by her neighbor j. Different choices for the J_{ij} are possible, inspired to percolation [33] or spin glasses models [34]. J_{ij} are assumed to be symmetrical in the present case but asymmetric J_{ij} could also be considered [35].

Noise ν_i represents a uniformly distributed shock to the agent's personal preference and is analogous to a temperature. $\epsilon(t)$ is an aggregate signal, uniformly accessible to all traders, following the arrival of news and acting as a driving field.

Each agent faces a specific transaction cost, modeled as a threshold which the agent's signal has to exceed to induce him to trade. Transaction costs generate a trade friction and lead a fraction of the agents to being inactive in any time period.

Each agent compares the signal (s)he receives with her individual thresholds, $\xi_i(t)$, and undertakes the decision:

$$\begin{aligned} S_i(t) &= 1 \quad \text{if} \quad Y_i(t) \geq \xi_i(t) \\ S_i(t) &= 0 \quad \text{if} \quad -\xi_i(t) < Y_i(t) < \xi_i(t) \\ S_i(t) &= -1 \quad \text{if} \quad Y_i(t) \leq -\xi_i(t) \end{aligned} \tag{2}$$

The $\xi_i(t)$ are chosen from a Gaussian distribution, with initial variance $\sigma_\xi(0)$ and zero mean, and are adjusted over time proportionally with movements in the stock price.

A consultation round to make decisions is allowed before trading takes place. Initially agents whose signal exceeds their individual thresholds make a decision to buy or sell according to eq. 2 and subsequently influence their neighbors' according to eq. 1. Traders decide sequentially and can revise past decisions on the basis of signals received from their neighbors. This process converges when no agent changes his decision. Once the decision making process is complete traders place their orders simultaneously.

Traders buy from or sell to a market maker who, at the end of every trading period, adjusts the stock prices according to the relative demand and supply and the overall trading volume.

The demand, $D(t)$, and supply, $Z(t)$, of stocks at time t are the numbers of buyers and sellers, respectively. The trading volume is $V(t) = Z(t) + D(t)$. After the transactions are complete the market-maker increments the stock price according to the rule

$$P(t+1) = P(t)\left(\frac{D(t)}{Z(t)}\right)^{\alpha} \tag{3}$$

where

$$\alpha = a\frac{V(t)}{L^2} \tag{4}$$

L^2 is the number of traders and represents the maximum number of stocks that can be traded in any time step. This rule describes the asymmetric reaction of market makers to imbalanced orders placed in periods of high versus low activity in the market and is consistent with the empirically observed positive correlation between absolute price returns and trading volume.

After the price has been updated the market volatility can be estimated as the absolute value of relative returns:

$$\sigma(t) = \mid log\ \frac{P(t+1)}{P(t)} \mid \tag{5}$$

Price changes lead to an adjustment of next period's thresholds, $\xi_i(t+1)$:

$$\xi_i(t+1) = \frac{P(t+1)}{P(t)} \xi_i(t) \tag{6}$$

This can be interpreted as an adaptive process such that the thresholds follow the local price trend. This will affect the subsequent volume of trade while conserving the symmetry in the probability of buying versus selling. If ξ_i arise from transaction costs, such as brokerage commissions, the adjustment process reflects a positive dependence of such costs on stock prices.

III. SIMULATIONS AND RESULTS

The outcomes of the model for different values of the parameters are simulated numerically. We focus our analysis on the volatility of returns.

The dimension of the lattice is set at $L = 100$. Each agent is initially given the same amount of stocks $N_i(0) = 100$ and of cash $M_i(0) = 100P(0)$, where $P(0) = 1$. The market maker is given a number of stocks, N_m, which is a multiple m of the number of traders (L^2) and an infinite amount of money.

The initial value of the thresholds' variance $\sigma_\xi(0) = 1$ and $\mu_\xi(0) = 0$. $\sigma_\xi(0)$ has been chosen close to its critical value in the $2-d$ RFIM at the same volume. As we said, in the quasi-static limit, the $2-d$ RFIM exhibits a non-equilibrium continuous phase transition driven by the amount of disorder. For large values of σ the spin flips are nearly uncorrelated, and the magnetization curve is smooth, formed by a sequence of small jumps. For small values of σ, on the contrary, the random fields are similar in different lattice sites, which results in highly correlated spin flips, and avalanches that may extend over the whole system. The passage from one kind of behavior to the other takes place at a critical value of the disorder. In our model σ_ξ changes with time changing consequently the level of correlation among the agents.

The coefficients A in eq.(1) is fixed at $A = 0.2$ and the individual noise signals ν_i are uniformly distributed in the interval $(-1, 1)$.

We will initially consider the arrival of aggregate news as the only mechanism which lead to a synchronized action among independent traders (we choose $J_{ij} = 0$ in eq.(1)).

Previous studies [20,21,40,41] on the effect of news on volatility autocorrelation have relied on a mechanism of sequential information arrival. In this contest it is assumed that some traders receive a particular piece of information early, some later and some not at all. The sequential arrival of investors to the market, some informed and some uninformed, induce a dynamic learning process that results in prices fully revealing the content of the private information. As opposed to simultaneous information arrival, when information arrives sequentially it generates a positive correlation between volume and absolute value of price changes and volatility clustering.

Here we will study the effects of uncorrelated news represented as a random sequence of positive and negative impulses of equal amplitude. Even if the information is simultaneously accessible to all agents in our model we are still able to generate correlations in the volatility of returns.

First of all we notice that with constant thresholds the effect of news arrival would be that of producing large price jumps of almost the same size whenever a news reaches the market (see fig.(1a)). The probability distribution of returns in this case presents two high peaks at large absolute return. When thresholds adjust dynamically, news of equal amplitude can generate price fluctuations of different sizes and fat tailed distribution of returns fig.(1b). When the frequency of news arrival is increased, volatility clustering also appears fig.(2).

Although the arrival of uncorrelated aggregate news can lead to volatility clustering, this is not the only channel through which large fluctuations in stock prices can be generated and sustained. The synchronization of traders' actions can be a pure consequence of communication and imitation effects across the agents.

We consider now the case $B = 0$, i.e. no information arrival, and $J_{ij} = 1$ for each pair of neighboring agents. In fig.(3) we plot (below) the stock returns $r(t)$ for the two scenarios: (a) fixed thresholds and (b) adjusting thresholds. The results are compared with the independent agent scenario (above).

The positive cross-correlation between price volatility and trading volume is shown in fig.(4).

Examples of realizations of the price history in various cases are shown in fig.(5). Observe that these do not display unrealistic behavior. Looking at them one would not be able to guess that one is associated with volatility clustering and the other is not.

Communication or news arrival helps synchronize traders' actions. The imitative behavior can spread through the system from one consultation round to the next, generating avalanches of different sizes in supply, demand and overall trading volume. The effect of larger trading volume and imbalance in supply and demand would be to increase volatility at one point in time.

However without the feedback on thresholds there would be no clustering effect. Volatility increases with both positive and negative large price fluctuation. Thresholds, on the other hand, increase when prices go up and decrease when prices go down. Through the effect of thresholds on trading volume subsequent volatility would increase (decrease) following an initially negative (positive) price change. If subsequent volatility were to increase following each direction of price change, the system would become unstable, while if it were to decrease in each case, any initial shock would be immediately dampened. Moreover, empirical results [18] indicate that volatility responds asymmetrically to positive versus negative price change. A direct interpretation of such an asymmetric change in trading volume to the direction of price change is also possible: when prices fall by a large amount agents are more likely to become aware and to react to subsequent news than when prices rise or stay constant. Indeed casual empiricism suggests that news of a collapse in stock prices is given disproportionate prominence in the media.

Budget constraints can affect the propagation of avalanches and possibly have an important role in generating a disequilibrium in the demand and supply. Agents can be prevented from buying or selling by, respectively, a shortage of money or stocks. This could reduce the influence of their neighbors. Nonetheless our results do not require a fine tuning of the agents' initial wealth or, indeed, other parameters. Preliminary results also show that, as opposed to other models [3,36–38], the volatility is not reduced and the clustering effect does not vanish at larger system sizes. However, in analogy with the RFIM [39], $\sigma_\xi(0)$ has to be appropriately chosen and to be decreased as the lattice size increases.

In fig.(6) we plot the autocorrelations function of return C_r and absolute return $C_{|r|}$

$$\begin{aligned} C_r(\tau) &= \quad < r(t)r(t+\tau) > - < r(t) >< r(t+\tau) > \\ C_{|r|}(\tau) &= < |r(t)||r(t+\tau)| > - < |r(t)| >< |r(t+\tau)| > \end{aligned} \tag{7}$$

as a function of the time lag τ.

Fig.(6) shows that while returns are not correlated, the autocorrelation function of absolute returns has a slowly decaying tail revealing the presence of long term memory. Inferring whether the decay of the autocorrelation function is exponential or power-law is nonetheless difficult from fig.(6). The nature of the long term correlations can be better investigated through the analysis of the variance of the cumulative returns and absolute returns. We construct the variables $\tilde{r}(t,\tau)$ and $|\tilde{r}(t,\tau)|$:

$$\begin{aligned} \tilde{r}(t,\tau) &= \tfrac{1}{\tau}\textstyle\sum_{i=1}^{\tau} r(t+i) \\ |\tilde{r}(t,\tau)| &= \tfrac{1}{\tau}\textstyle\sum_{i=1}^{\tau} |r(t+i)| \end{aligned} \tag{8}$$

where the sum is taken over non overlapping intervals. The quantity we are interested is the variance of these new variables as a function of τ. It can be shown [14,15] that if the correlation functions of absolute returns has a power-law decay $C_{|r|}(\tau) \sim \tau^{-\beta}$ with an exponent $\beta < 1$ then the $var\ (|\tilde{r}(t,\tau)|)$ is a power-law with the same exponent:

$$var(|\tilde{r}(t,\tau)|) \sim \tau^{-\beta} \tag{9}$$

On the contrary if the $|r(t)|$ are uncorrelated or short term correlated we would find

$$var(|\tilde{r}(t,\tau)|) \sim \tau^{-1}. \tag{10}$$

In fig.(7) we compare the standard deviation of cumulative returns and absolute cumulative returns. In the situation where no thresholds and no couplings among the agents are introduced (see fig.(7a)), as expected $\beta \simeq 1.$, in both cases. On the contrary, when agents interaction and adjusting thresholds are integrated in the model, $\beta \simeq 1.$ for the cumulative returns and $\beta \simeq 0.2$ for the absolute cumulative returns (see fig.(7b)). Empirical studies [10,11,14,15,42] have estimated $\beta \simeq 0.38$ for the absolute returns autocorrelation in the NYSE index and $\beta \simeq 0.39$ for the USD-DM exchange rate.

IV. CONCLUSIONS

This paper has outlined a mechanism which can explain certain stylized facts of stock market returns. According to our model synchronization effects, which generate large fluctuations in returns, can arise purely from communication and imitation among traders, even in the absence of an aggregate exogenous shock.

Many interesting questions which have arisen in other studies could also be addressed in the context of our model. One of these is how the trading mechanism affects the distribution of wealth among the traders.

Given an initial wealth distribution, how does it change with time as a result of trading mechanisms? Previous studies suggest [43,44] that, in a stationary state, the distribution of wealth follows a well defined power law in accord with the Zipf law [45]. From a preliminary analysis of traders' wealth in our model we observe the emergence of power law distributions but a longer simulation is needed to reach a stationary situation.

Recent studies [46] suggest that crashes have a characteristic log-periodic signature in analogy with earthquakes and other self-organizing cooperative system [47]. Another interesting question is whether precursor patterns and after shock signatures of financial crashes can be identified in the simulated price histories of our model.

ACKNOWLEDGEMENTS

I would like to thank E. Scalas, G. Susinno, V. Koulovassilopoulos, D. Farmer, T. Lux, S. Solomon, D. Stauffer for interesting discussion and suggestions and especially C. Hiemstra and S. Jafarey for valuable comments.

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FIGURES

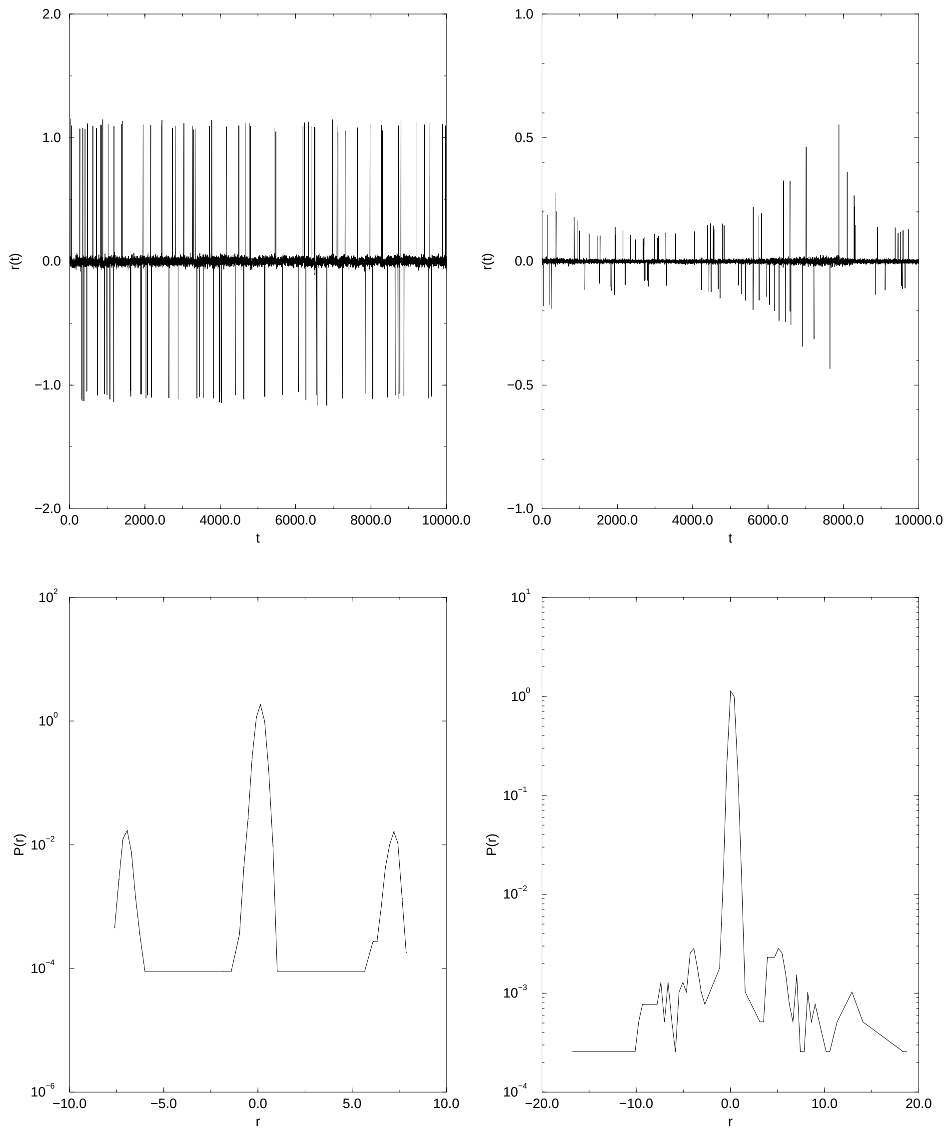

FIG. 1. (a)Returns and their distributions in the model with news arrival (a) fixed thresholds $J_{ij} = 0, B = 0.10, p_n = 0.01$; (b) adjusting thresholds $J_{ij} = 0, B = 0.10, p_n = 0.01$;

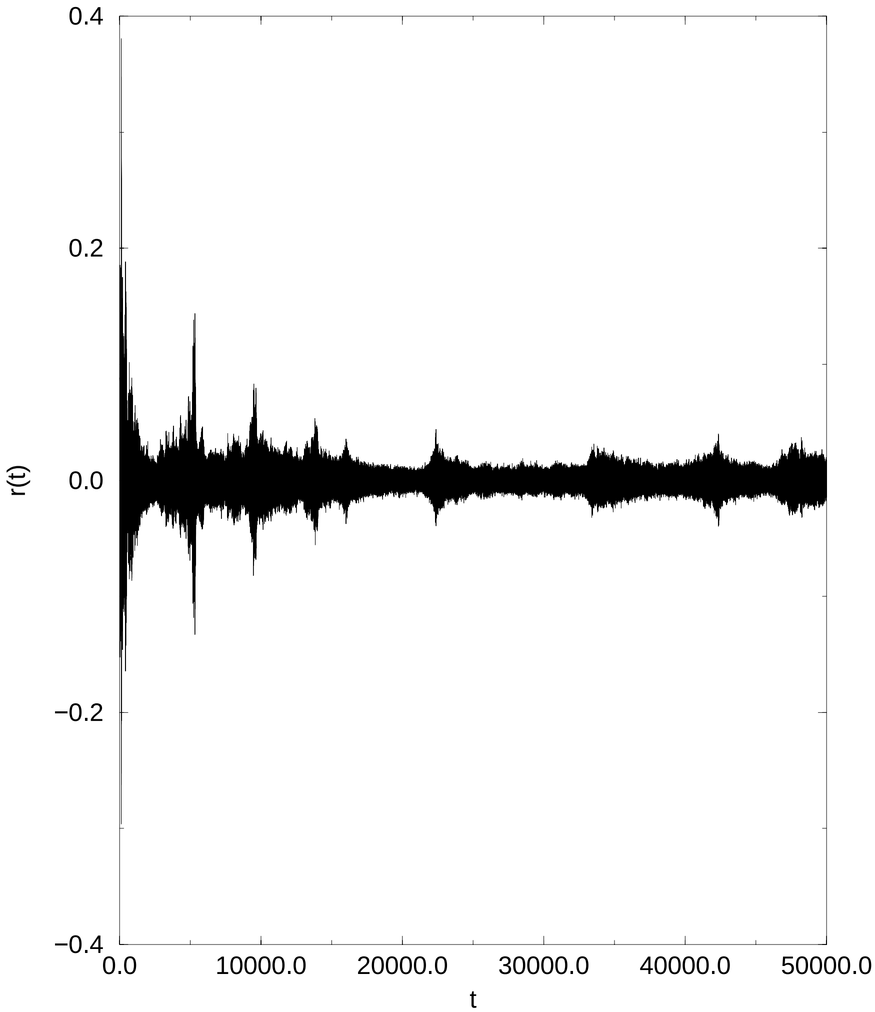

FIG. 2. Volatility clustering in the model with news arrival and adjusting thresholds, $J_{ij} = 0, B = 0.05, p_n = 1$;

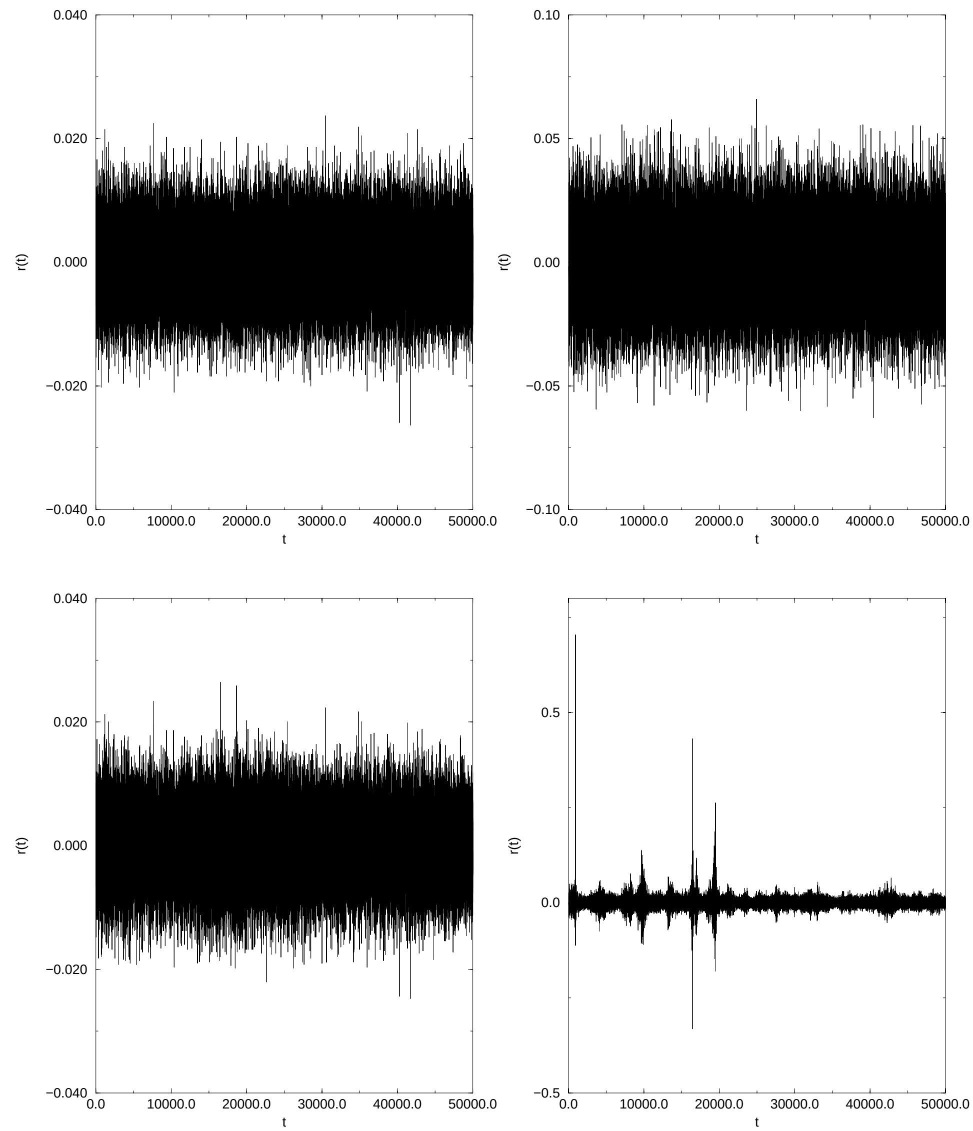

FIG. 3. Returns $r(t)$ for, above, (a1) $\sigma_{\xi(t)} = 1$, $J_{ij} = 0$, (a2) $\sigma_{\xi(t)} = 1$, $J_{ij} = 1$; below, (b1) adjusting $\sigma_{\xi(t)}$, $J_{ij} = 0$, (b2) $\sigma_{\xi(t)}$, $J_{ij} = 1$

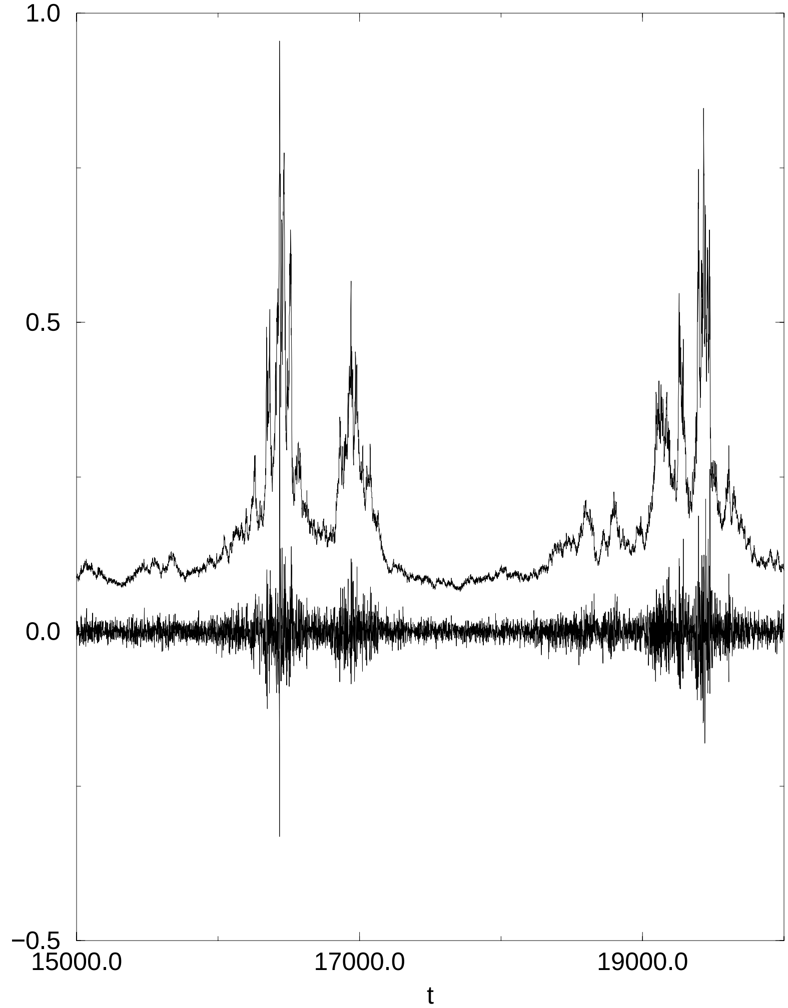

FIG. 4. Price return (below) and percentual trading volume (above) for $J_{ij} = 1$.

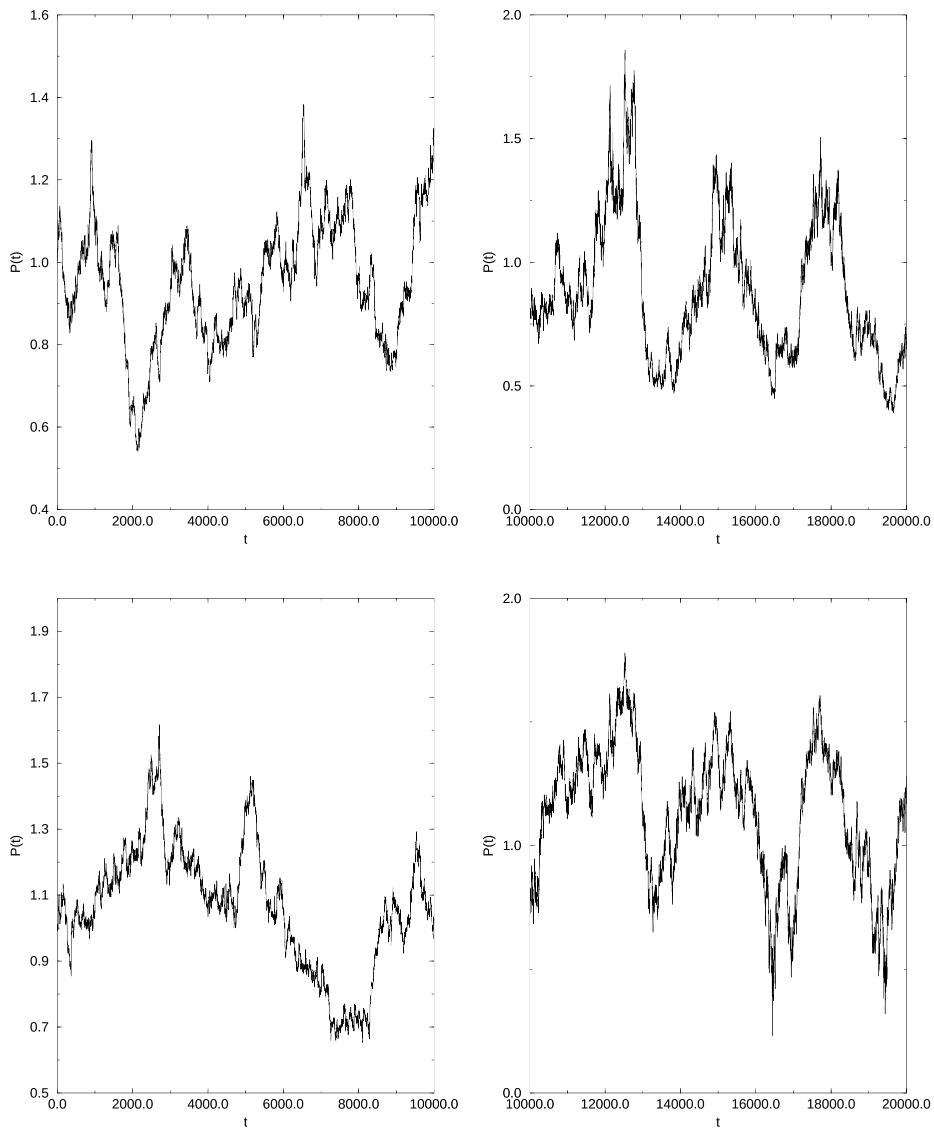

FIG. 5. Price history for above with fixed thresholds and below with adjusting thresholds (a) $J_ij = 0, B = 0.03$, $J_ij = 1, B = 0$.

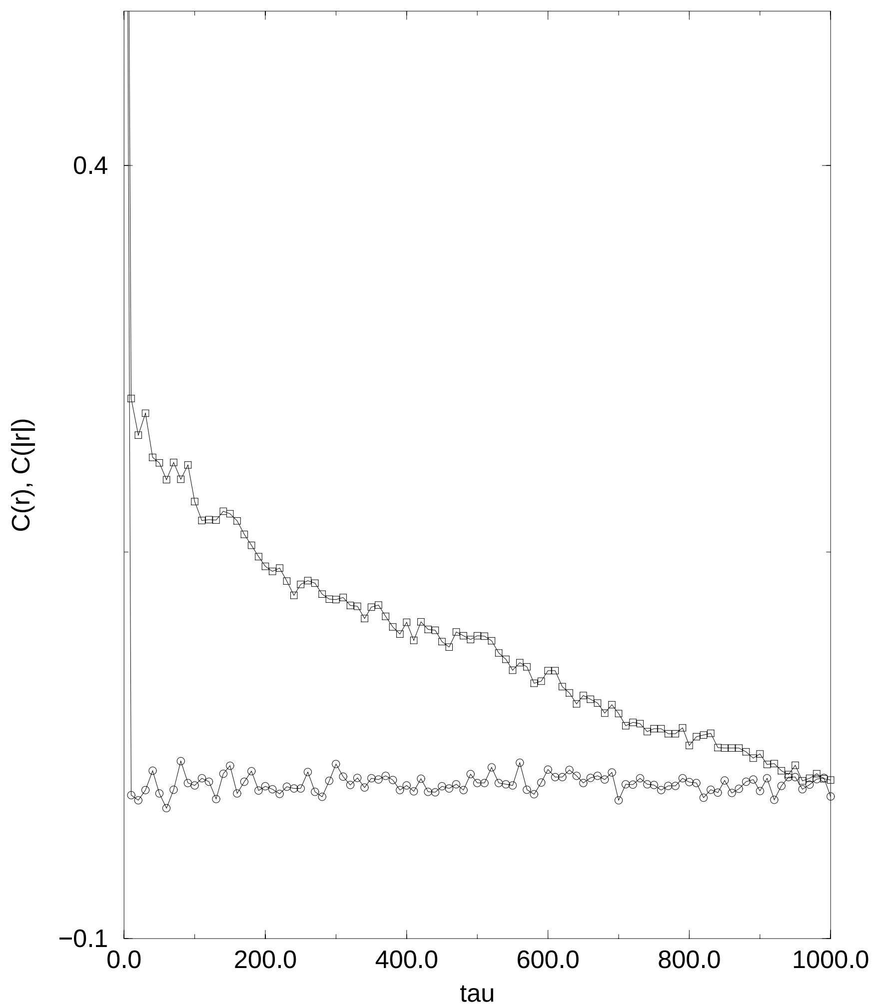

FIG. 6. Autocorrelations function of returns (circles) and absolute returns (squares).

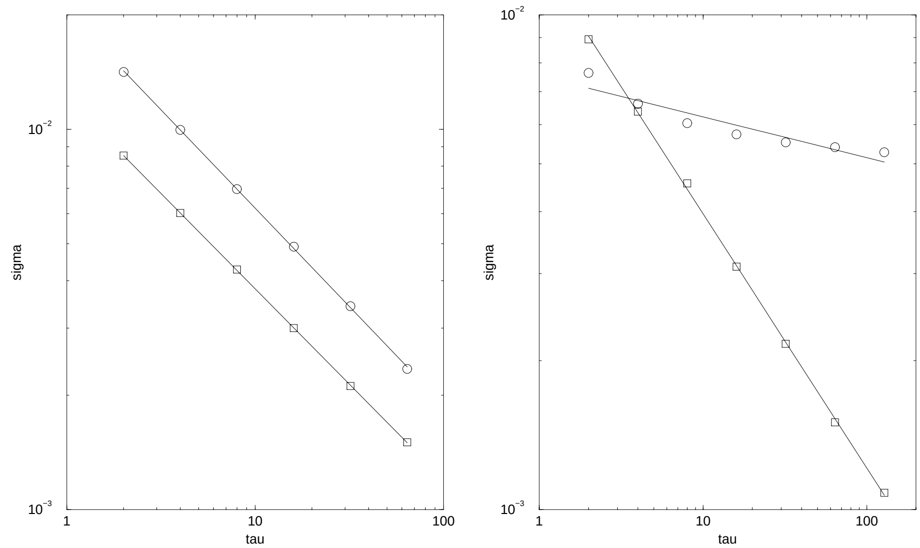

FIG. 7. Standard deviation of cumulative return (squares) and cumulative absolute return (circles) (a) $\sigma_\xi(0) = 0, J_{ij} = 0$, (b) $\sigma_\xi(0) = 1, J_{ij} = 1$