"The Evolution of Trading Rules in an Artificial Stock Market"

by

Mark Howard
Ph.D. Candidate in Economics
University of Massachusetts, Amherst
email - mmh@econs.umass.edu

First version December 8,1998
This version June 25, 1999

Abstract

This paper applies evolutionary modeling to expectation formation of an asset's price. As a first step I consider a population of n investors each of whom take on one of two possible cultural variants. Every individual is a potential role model for all other individual and can pass on their variant with a certain probability determined by the relative return to being that type. Different types of traders operate on different 'models' which forecast future price and dividend movements. The two basic types being traders being those who follow the fundamentals suggested by the CAPM model and those who follow technical trading rules such as buy if the price is above it's 50 day moving average and sell if it is below. I show that given these two types of simple traders prices can fluctuate between periods of low volume and volatility and periods of high volume and volatility. Results indicate that given a random walk fundamental valuation, as the random fluctuations increase in magnitude, technical trading can become more profitable than fundamental trading, and for a period dominate the market.

** Presented at the CEF99 conference at Boston College on June 25th, 1999

This paper applies evolutionary modeling to expectation formation of an asset's price. The need for a new vision of asset price formation arises from the failure of the standard academic model exemplified in the efficient-market model to explain observed price data. In particular the efficient-market model fails to explain the periods of high volume and volatility as well as periods of tranquillity, apparent in actual stock markets. The model presented here envisions different types of traders operating on different 'models' which forecast future price and dividend movements. The two basic types being traders being those who follow the fundamentals suggested by the CAPM model and those who follow technical trading rules such as sell if the price is above it's 50 day moving average. I aim to show that given these two types of simple traders prices can fluctuate between periods of low volume and volatility and periods of high volume and volatility.

The standard academic treatment of asset pricing usually begins with the Capital Asset Pricing Model where an individual investor chooses a portfolio of assets so as to maximize her utility. In doing so the she is concerned only with the expected value and the standard deviation of her prospective return. One assumption of the model is that expectations regarding future outcomes are homogeneous. That is all investors have identical estimates of the means and variances of returns on all potential assets and these are correct. Assuming homogeneous expectations which are correct and unchanging would in fact preclude a market if not for differing rates of time preference. This runs counter observed market activity, that is, the reason for market activity is that people have different expectations. Some investors think it is time to buy others think it is time to sell. Some are right and some are not. In a world such as the one envisioned by the CAPM it is hard to imagine anything but a market for new issues since, "it is implicit in the theory that once portfolios are determined as being mutually optimal for all investors they will be held for ever." (Vickers pg.76) Such a model clearly cannot explain the continual disappointment of expectations of a large number of investors. Additionally it is not capable of explaining the booms and busts we see on a regular basis. Empirically this model does not hold up.

One of the simplest and in some sense strictest implications of the foregoing is that in equilibrium no arbitrage is possible. This is one form of the efficient markets hypothesis which would imply very low trading volume and volatility. There have been many attempts to empirically test the efficient markets hypothesis. The Test of the simple version(the no arbitrage condition) is easy since it implies virtually zero volatility and volume it can easily be rejected since actual stock market prices show periods tremendous volume and volatility. The more sophisticated tests follow Samuelson's(1965) vision of efficient capital markets which implies prices should follow a Martingale process, which would permit a certain amount of volatility. LeRoy (1989) has argued even for the more plausible versions "asset prices appear to be more volatile than is consistent with the efficient-markets model"

In The General Theory J.M. Keynes argued that the stock market was very much like a beauty contest where the individual who can pick the prettiest face wins. But the prettiest face is determined by the average opinion of all who play. As Keynes said,

> "professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole ."(pg. 156)

This suggests that investors are rewarded for their ability to conform to the societal norm with regard to their expectations. That is the investor who best assimilates the culture of the market will prosper. This leads to the notion that the entire process can be seen as one of cultural evolution where individual investors 'inherit' the expectations they hold in the present from individuals encountered in the past, including themselves.

Samuelson vs Sorros

Actual traders fall into two categories. There are those who follow the underlying fundamentals. They look at P/E (price to earnings) ratios and attempt to predict future earnings with the notion that any stock has an fundamental value based these future expected returns. These fundamental analysts hope to make money by buying when the price is below the fundamental value and selling when it is above. Alternatively there are technical analysts. These traders believe that regular patterns appear in the sequence of prices and therefore trading rules based purely on past prices can lead to profits. One technical trading rule is buy if the price is above the 100 day moving average and sell if it is below.

Arthur et. al. have examined this idea using a artificial stock market. They run an artificial market consisting of 25 utility maximizing agents each analyzing 100 linear expectational models (APT equations). Each expectational model employs 12 data bits or market descriptors, which are equal to 1, 0 or # (not used). The predictors are selected randomly for recombination . The first six bits reflect fundamental information , like the P/e ratio is greater than 20. The last six reflect technical information like the price is x% above the 100 day moving average. In the next period each learning agent employs the best predictor model among the 100 she analyzes this period. They show two different “regimes” possible. One is under low learning rates in which case “the market price , in these experiments, converges rapidly to the homogeneous rational expectations value" which is the underlying fundamental value. As a higher rate of learning occurs they see the "appearance of bubbles and crashes [which] suggest that technical trading,… has emerged in the market." (pg 29) The simulated results duplicate the "look" of actual market data but do not draw out much in the way of an analytic model. This paper hopes to repeat their results with a simple analytic model that can then be used to explore the basic laws of motion of such a evolving process.

The basic model

As a first step consider a population of n investors each of whom take on one of two possible cultural variants. This is a simple dichotomous trait. Each is either a Fundamentalist or Techie with the obvious interpretation thereof. Every individual is a potential role model for all other individual. Thus in the next period they can pass on their variant with a certain probability. It might make sense to choose role models who were successful in the previous period.

$\rho_t \equiv$ % of traders that play the technical strategy in time period t

$1-\rho_t \equiv$ % of traders that play the fundamental strategy in time period t

z_t = % of shares held by techies in time period t

$1-z_t$ = % shares held by fundamentalist in time period t

N = total number of traders

S = total number of shares

MA_x = x period moving average of the price P

$E(P_{t+1}^F) \equiv$ price expected by the fundamental traders in time period t

$E(P_{t+1}^T) \equiv$ price expected by the technical traders in time period t

W_t^T = Total techie wealth in period t (at the end of period t)

W_t^F = Total fundamentalist wealth in period t (at the end of period t)

$C_t^{\ T}$ = Total cash held by techies in period t

$C_t^{\ F}$ = Total cash held by fundamentalist in period t

w_t^T = average techie wealth $= W_t^T/\rho N$

w_t^F = average fundamentalist wealth $= W_t^F/(1-\rho)N$

Replicator dynamics.

I model the change in the population frequencies of traders as a function of each type's average wealth level. Assuming that at the end of each period if the trader met a trader of the other type they can compare wealth. If the other type has a higher wealth then the trader will change her type taking her existing (average) wealth both cash and shares with her . Assuming the different types never have exactly the same average wealth, the number that change type each period will be equal to the number of meeting (N/2) time the probability that the two traders are of different type ($\rho_t(1-\rho_\tau)$). Dividing the result by the number of traders N yields the portion of the population switching.

If $W_t^T > W_t^F$

$$\Delta\rho = + ((N/2)\rho_\tau(1-\rho_\tau))/N = \rho_t(1-\rho_\tau)/2 \text{ and} \tag{1a}$$

if $W_t^T < W_t^F$

$$\Delta\rho = - ((N/2)\rho_\tau(1-\rho_\tau))/N = - \rho_t(1-\rho_\tau)/2 \text{ and} \tag{1b}$$

$$\rho_{t+1} = \rho_t + \Delta\rho \tag{2}$$

Total wealth of each type is equal to the total cash held by that type plus the total shares held by that type times the current price. For each

$W_t^T = C_t^{\ T} + P_t z_t *S$

$W_t^F = C_t^F + P_t(1\text{-}z_t)*S$

For the techies the average wealth in time t is

$w_t^T = W_t^T/N\rho_t = (C_t^T + P_t z_t *S)/ N\rho_t$ (3a)

$w_t^F = W_t^F/N(1\text{-}\rho_t) = (C_t^F + P_t(1\text{-}z_t)*S)/ N(1\text{-}\rho_t)$ (3b)

Which means the proportion of shares held by techies (z) must be specified. This can be done as follows. Recall trade only occurs if the two agents are of the opposite type and the one who wants to sell has the share to sell. Now after a trade the two traders compare there portfolios. The one whose portfolio (the average of that type) is valued less will switch to the other type.

The change in shares per period can be calculated as follows. If the techies expected price is greater than the fundamentalist then the techie will buy a share from the fundamentalist if it is available. How many techie purchases (TPP) will take place per period?

TTP = (# of meetings) * (prob. of two different types) * (prob. type F has the share)

$TTP = (N/2)*(\rho_\tau(1\text{-}\rho_\tau))*(((1\text{-}z_t)*S)/(1\text{-}\rho_\tau)*N) = \rho_\tau(1-z_t)*S/2$

This is the total shares bought(sold) by techies from(to) fundamentalists in period t. Dividing this result by the total outstanding shares yields the change in the percent of shares held by techies.

$\Delta z = (N/2)*(\rho_\tau(1\text{-}\rho_\tau))*(((1\text{-}z_t)*S)/(1\text{-}\rho_\tau)*N)/S = \rho_\tau(1-z_t)/2$ (4a)

if $E(P_t^T) > E(P_t^F)$

and

$\Delta z = \text{-}(N/2)*(\rho_\tau(1\text{-}\rho_\tau))*(((z_t)*S)/(\rho_t)*N)/S = \text{-}(1\text{-}\rho_t)z_t/2$ (4b)

if $E(P_t^T) < E(P_t^F)$

and finally

$\Delta z = (N/2)*(\rho_\tau(1\text{-}\rho_\tau))*(((1\text{-}z_t)*S)/(1\text{-}\rho_\tau)*N)/S = \rho_\tau(1-z_t)/2$ (4a) if $E(P_t^T) > E(P_t^F)$

and

$$\Delta z = -(N/2)*(\rho_\tau(1-\rho_\tau))*(((z_t)*S)/(\rho_t)*N)/S = -(1-\rho_t)z_t/2 \quad (4b)$$

if $E(P_t^T) < E(P_t^F)$

and finally

$$z_{t+1} = z_t + \Delta z + (\Delta\rho z_t \text{ if } W_t^T < W_t^F, \text{ and } \Delta\rho(1-z_t) \text{ if } W_t^T > W_t^F) \quad (5)$$

where the last term represents the percent of shares switched as traders who change type.

To specify equations 1a and 1b above we need to know the cash and shares held by each type of trader. The cash held by techies can be specified as

$$C_{t+1}^{\ T} = C_t^{\ T} - \Delta z P_t +(- \Delta\rho\, C_t^{\ T} \text{ if } W_t^T < W_t^F \text{ and } + \Delta\rho\, C_t^{\ F} \text{ if } W_t^T > W_t^F) \quad (7)$$

where the last term represents cash switched as traders change type.

Pricing

Assume traders are randomly paired each period. Since there are only two types and since trade is based on differing expected prices they only trade when they meet a trader of the opposite type. If they are of opposite types then if the one who has the lower expected price has the stock and the other does not then they trade. They split the difference in their prices. Assuming that at least one trade takes place. The resulting price will always be ,

$$P_{t+1} = .5\, E(P_{t+1}^{\ T}) +.5\, E(P_{t+1}^{\ F}) \quad (8)$$

Assume the fundamentalist trader has the following price expectation,

$$E(P_{t+1}^{\ F}) = F_t = F_{t-1} + \varepsilon_\tau \quad (9)$$

where F = (d/r) where d is the fixed perpetual dividend and r is the risk free rate of return and ε_τ is a uniform random variable with mean zero.

Model 1 - A simpleton techie

$E(P_{t+1}{}^{T}) = P_t + P_t - P_{t-1} = 2P_t - P_{t-1}$ (10)

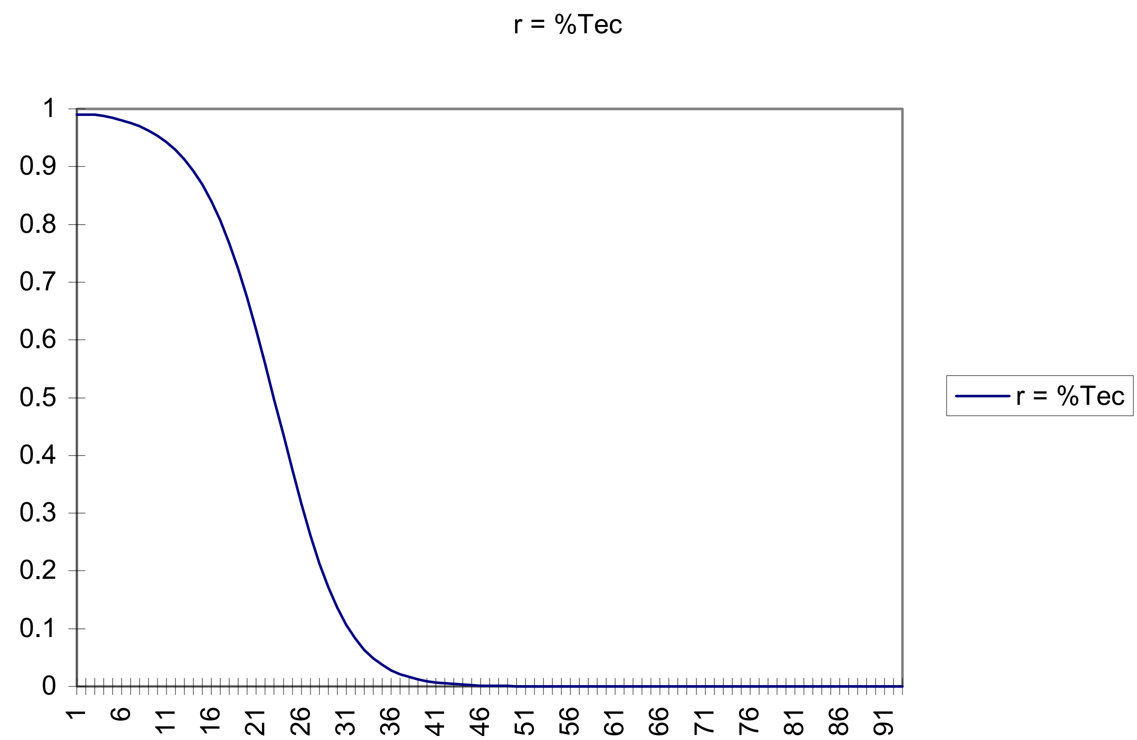
This says that that the simple techie assumes that what ever happened last period will happen again this period. So her expected price next period is the current periods price plus the difference between the current period and the last period.

Substituting (9) and (10) into (8) yields

$P_{t+1} = .5(2P_t - P_{t-1}) + .5F_t$ (11)

Before I attempt a more rigorous proof here is my intuition on replication. Fundamentalist will on average buy stocks when $P_t < F_t$ and sell when $P_t > F_t$. . Since P_t is equally likely to be greater than or less than F_t , on average it would appear that the fundamental strategy can not lose and therefore would eventually be held by all traders. Thus on average the changes to the cash portion of the traders portfolio goes up for Fundamentalist and down for Techies then on average the portion of Techies will steadily decrease. As more and more traders hold the same expectation fewer and fewer trades occur with volume going towards zero. This confirms the efficient markets (rational expectations/ CAPM) result.

I ran this simulation on Excel. The results Confirming the conclusion above. This can be seen in the following chart graphing the proportion of techies in the population over time.



Model 2 a smarter techie

Assuming the same pricing equation and Fundamentalist rule as above, in this model the techie follows the rule,

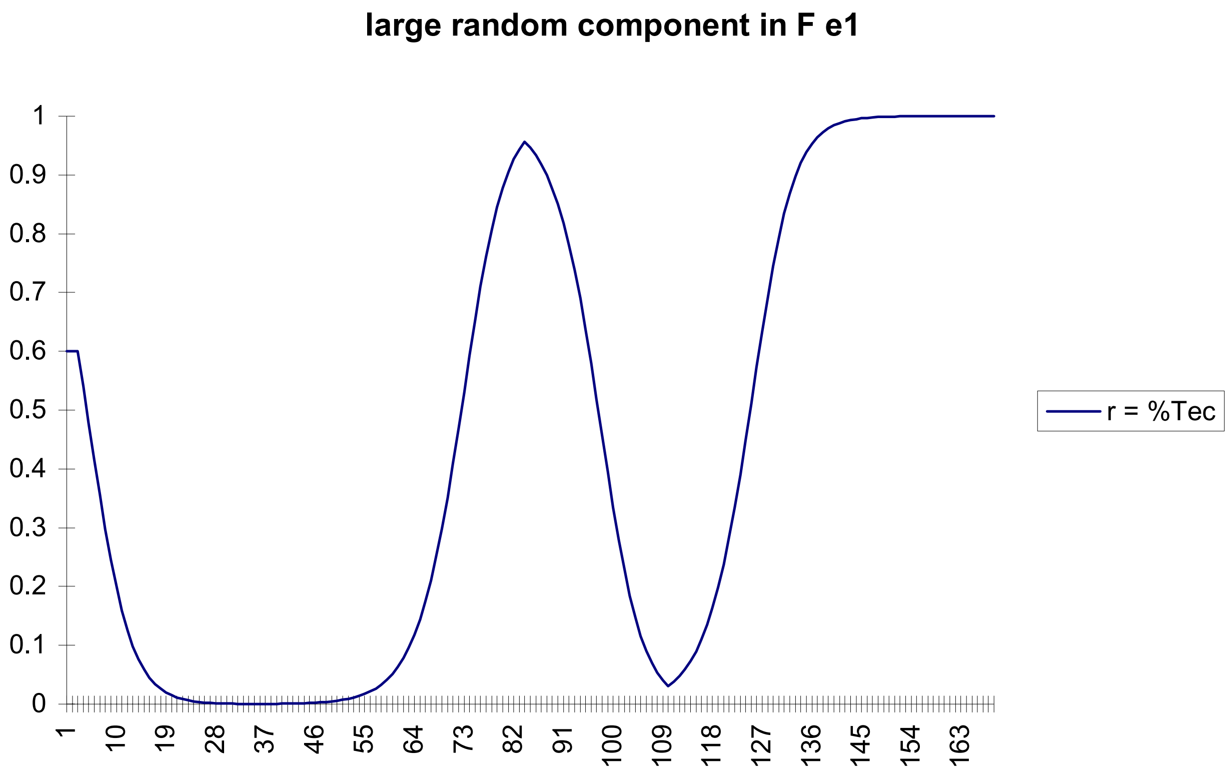
$$E(P_{t+1}^{T}) = P_t + P_t - P_{t-1} = 2P_t - P_{t-1} \quad \text{if } (1-y)MA_x > P_t > (1+y)MA_x$$

$$= MA_x \quad \text{otherwise}$$

where $0<y<1$

I ran this model with y=.2 and x = 10.

The problem is while this techie is smarter she still will face the same fundamentalist who has the same advantage, always buying when $P_t < F_t$ and selling when $P_t > F_t$.. And again since the price is equally likely to be greater or less than F_t the fundamentalist can't lose and therefore the techie still can't win. This is the case when the random component of the fundamentalist expected price is small. However as the random component gets larger the situation changes so that the population frequency can vary drastically without eliminating techies all together. For example



This mirrors Arthur et. al.'s results. The problem with this is the random part of F is very large 50% of the value of F. Furthermore this technical trader is really an economist at heart. He expects the price to revert to it's mean. A real world techie would follow a trading rule such as buy if the 5 day moving average cuts above the 20 day moving average and sell when the 5 day moving average cuts below the 20 day moving average. This leads to a third model of the profitable techie.

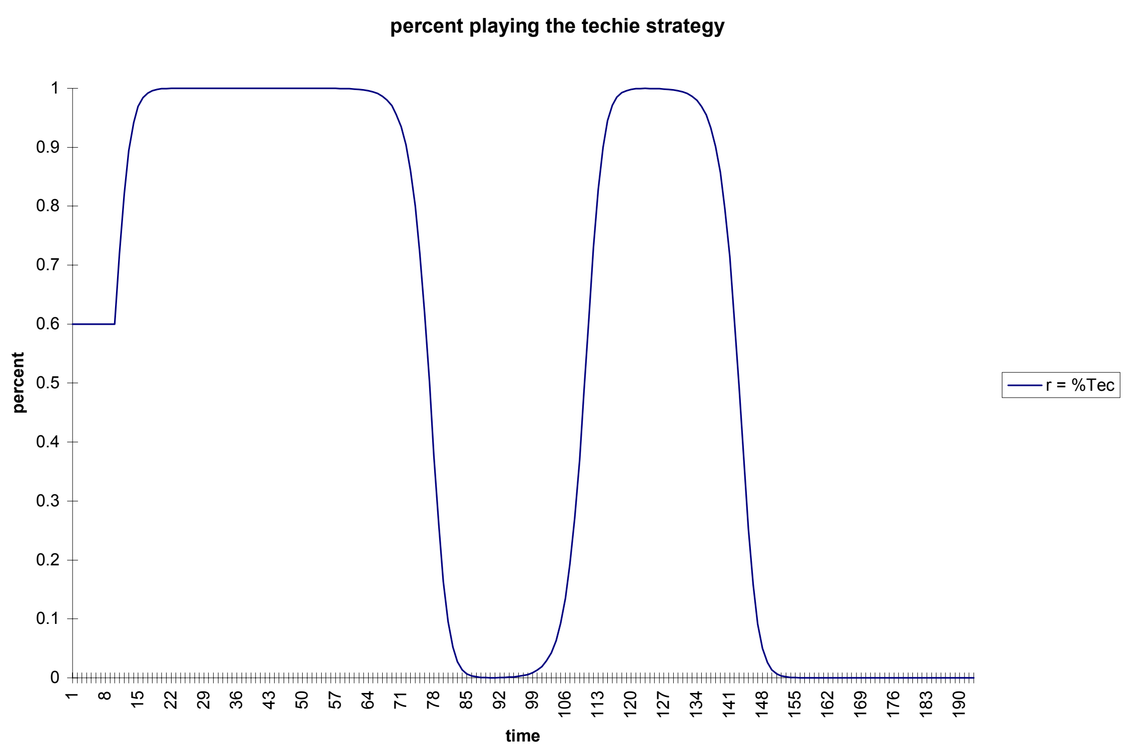
Model 3 a profitable techie

Assuming the same pricing equation and Fundamentalist rule as above, in this model the techie follows the rule,

$E(P_{t+1}^{T}) = P_t + | P_t - P_{t-1}|$ if $MA5_t > MA20_t$ and

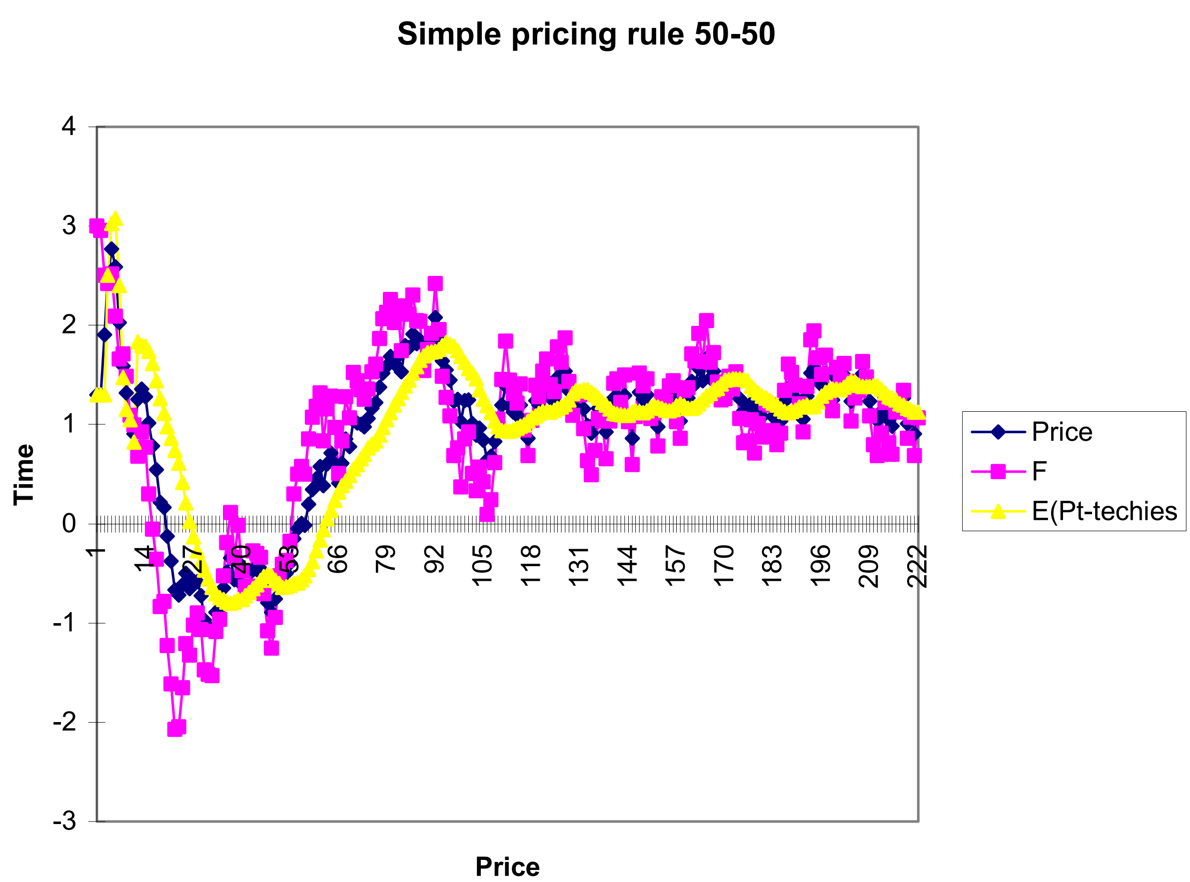
$= P_t - |P_t - P_{t-1}|$ if $MA5_t < MA20_t$

where MA5 and MA20 are the 5 and 20 period moving averages respectively. This says the techie assumes the price will rise if the 5 day moving average is greater than the 20 day moving average and fall when the 5 day moving average is below the 20 day moving average. In this case the random component of F averaged 10% of the total. In most runs of this simulation techies outnumbered fundamentalist for many periods. The results of one such simulation is shown below.



In the next model I want to add another type, a true noise trader. This type would predict the price moves as a random variable independent from the fundamentals. Also traders in the model above are infinitely lived, which is not realistic. Having traders with a "life span" would mean a certain portion of traders would have to leave the market every period regardless of the price, with new ones entering. This I think would allow techies to do better with a smaller random component.

Recall the pricing equation was independent of population frequencies and results in a price that basically lags behind the fundamental value. For example in the second simulation above the price and expected prices over time are,



The problem I have with this scenario is that the pricing mechanism dose not take into account the differing numbers/proportion of buyers and sellers in each time period. Suppose

$E(P_{t+1}{}^{T}) = P_t + P_t - P_{t-1} = 2P_t - P_{t-1} < F_t = E(P_{t+1}{}^{F})$

so Fundamentalist want to buy and Techies want to sell. Fundamentalist would prefer the Techies price and Techies would prefer the Fundamentalist price. If each trader had instant access to the market then for each Techie wanting to sell there would be $(1-\rho)/\rho$ Fundamentalist wanting to buy. If the price were to reflect the relative numbers of buyers for each seller for say $\rho = 1/3$ then there would be two buyers for each seller. It seem reasonable in this case that rather than splitting the difference in there expected prices the seller being on the short side of the market should do better than that. Since there are in this case two buyers for every seller suppose the price is the average of twice the sellers preferred price plus the buyers. This type of pricing splits the difference in the two prices according to the population frequencies implying,

$P_{t+1} = \rho\, E(P_{t+1}{}^{T}) + (1-\rho)\, E(P_{t+1}{}^{F})$.

While I think this pricing rule is more realistic than the previous one I need a better justification for it. (See Farmer also at this conference)

Once that is done the price dynamics that result from this equation are much more interesting than the previous one. Also I think it makes sense to borrow from Bowles (1998) and have the probability that a trade occurs be a function of the difference between the traders price. This will result in a higher volume of trading as the volatility increases.

References

Arthur, B. W., Holland, J.H., LeBaron, B., Palmer, R., and Tayler P. "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market" in The Economy as a Evolving Complex System II Eds. Durlauf, Arthur, and Lane 1997

Bowles, Samuel "Spontaneous Order and the Evolution of Behaviors" lecture notes 1998

Farmer, J. Doyne "Market Force, Ecology, and Evolution" 1998 presented, at the CEF99 conference at Boston College on June 25th, 1999

Keynes, J.M (1953) The General Theory

LeRoy, Stephen (1989) "Efficient Capital Markets and Martingales" JEL December 1989 pg 1596

Sharpe, William (1964), "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk.", Journal of Finance 19, no. 3:425-42

Vickers (1994), Economics and the Antagonism of Time