Herd behavior and aggregate fluctuations in financial markets

Abstract

We present a simple model of a stock market where a random

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Keywords: communication, market organization, random graphs.

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Empirical studies of the fluctuations in the price of various financial assets have shown that distributions of stock returns and stock price changes have heavy tails that deviate from the Gaussian distribution especially for intraday time scales. These facts are characteristic of a significant excess kurtosis persist over the distribution. Furthermore, fat tails that deviate from the Gaussian distribution of stock returns and stock price changes have been shown to correspond to collective phenomena such as crowd effects or market herding behavior in financial markets. What are the implications of herding for relations between market variables? How does the presence of herding modify the distribution of returns? Which are the statistical properties of market demand due to herding and interaction for the statistical properties of market demand? Are there novel theoretical insights on the implications of these phenomena? These are some of the questions which have motivated our study.

The aim of the present study is to examine, in the framework of a simple model, how the existence of herd behavior among market participants may generically lead to large fluctuations in the aggregate excess demand, described by a heavy-tailed non-Gaussian distribution. Furthermore, we explore how empirically measurable quantities such as the excess kurtosis of returns and the average order arrival may be related to each other in the context of our model. Our approach provides a quantitative link between the two sets of our model. Our approach provides a quantitative link between the two sets of market returns and the average order arrival at each order in the context of our model. Our approach provides a quantitative link between the two sets of market returns and the average order arrival at each order in the context of our model.

The article is divided into four sections. Section 1 revises well known empirical studies of the fluctuations in the price of financial assets. Section 2 presents provision for the calculation of various financial models proposed to account for the empirical and theoretical work on clustering and interaction in financial markets. Section 3 presents provision for the calculation of various financial models proposed to account for the empirical and theoretical work on clustering and interaction in financial markets. Section 4 presents provision for the calculation of various financial models proposed to account for the empirical and theoretical work on clustering and interaction in financial markets.
The heavy-tailed nature of asset return distributions

It is well known that in the presence of heteroskedasticity, the unconditional distribution of returns will have heavy tails. In most models based on conditional heteroskedasticity, the process of return generation is assumed to be conditionally Gaussian; the shocks are "locally" Gaussian and the non-Gaussian character of the unconditional distribution is an effect of aggregation. Well known examples are Mandelbrot's stable Paretian hypothesis, the mixture of distributions hypothesis, and models based on conditional heteroskedasticity. Many statistical models have been put forth to account for the heavy tails of the distribution of asset returns. Many of these models assume that daily returns are standard normal variables, while the distribution of returns over a longer horizon is some exponential decay for most assets. The study of the tails of the distribution shows an exponential decay for most assets. The heavy-tailed nature of asset return distributions is quantitatively reflected in that developed from a normal distribution. This is graphically reflected in random individual demands in a market. Section 3 presents analytical results, Section 4 discusses the results in economic terms, and Section 5 interprets the results in economic terms.
In short, although heteroskedasticity and time deformation partly explain
the fluctuations in trading volume or number of trades,
the returns are conditionally normal. Thus, conditional volatility is the
important phenomenon to explain.

Although conditional normality is not normally
observed in the empirical data, several alternative models have been
proposed. One such model is the conditional autoregressive heteroskedastic
model (GARCH). This model allows for conditional heteroskedasticity, which
is not always observed in the empirical data. The GARCH model is
formulated as a linear combination of past squared returns and past
volatility, which allows for feedback effects. The GARCH parameters
are estimated using maximum likelihood estimation.

A second approach is to model returns using a subordinated process, which
is a stochastic time change process. This process is characterized by
a transformation of the underlying process, such as a Brownian motion or
a Poisson process. The resulting process is called a subordinated process,
and it can be shown that the subordinated process has heavy tails and
excess kurtosis.

A third approach, first advocated by Clark, is to model stock returns as
a subordinated process, specifically a subordinated Brownian motion.

In summary, although heteroskedasticity and time deformation partly explain
the returns, conditional normality is not always observed. Several alternative
models have been proposed to explain this phenomenon, including
the GARCH model and subordinated processes.
Accounting for these effects, one is left with an important residual kurtosis in the resulting transformed time series. Moreover, these approaches are not based on any particular model of the market phenomenon generating the data that they attempt to describe. Recent works by Bäck, Paczuski, and Shubik and Zhang have led to explain the heavy tailed nature of return distributions by modeling the communication structure between market agents as a random graph process. We present here an alternative approach which, by modeling the communication structure between market agents as a random graph, proposes a simple mechanism accounting for some non-trivial statistical properties of stock price fluctuations. Although much more rudimentary and containing fewer ingredients than the model proposed by Bäck, Paczuski, and Shubik, our model allows for analytic calculations to be performed, thus enabling us to interpret in economic terms the role of each of the parameters introduced. The basic intuition behind our approach is simple: Interaction of market participants through imitation can lead to large fluctuations in aggregate demand, leading to heavy tails in the distribution of returns. Herd behavior in financial markets Anumber of recent studies have considered mimetic behavior as a possible explanation for the excess volatility observed in financial markets. The existence of herd behavior in speculative markets has been documented by a certain number of studies. Schwert and Stein discuss evidence of the existence of herd behavior in speculative markets. However, the phenomenon of the excess volatility observed in financial markets is not explained by these approaches. Therefore, these approaches are not accounting for these effects, one is left with an important residual kurtosis.
Herding in the behavior of fund managers, Grinblatt et al.

Report herding in mutual fund behavior while Trueman and Welch show evidence for herding in the forecasts made by financial analysts.

On the theoretical side several studies have shown that in a market where there is no alignment of beliefs among traders, herding behavior is not necessarily irrational. The case of joining it will also be a matter of opinion, the core of joining it will also be a matter of opinion.

The approach posed in this paper is different from both approaches. The observed for stock returns are obtained a heavy-tailed unimodal distribution centered at zero such as collective market phenomena such as crashes or panics. In neither case does the Gaussian distribution when the imitation is weak, or a bimodal distribution with non-zero modes, which Grinblatt et al. consider invalid, reappear. The assumption seems unrealistic in the case of financial markets. The assumption seems unrealistic in the case of joining it will also be a matter of opinion. The case of joining it will also be a matter of opinion. This assumption seems unrealistic in the case of financial markets.

Various models of herd behavior have been considered in the literature, the most well-known approach being that of Panfili and Bilgihan.

Bayesian process, global information cascades, and the model that assume a multi-modal distribution of initial beliefs between agents in which two agents have the same tendency to imitate each other. In terms of aggregate variables, this model leads either to a Gaussian distribution when the imitation is weak, or to a bimodal distribution with non-zero modes, which Orléan considers invalid. Orléan considers invalid. Orléan considers invalid. Orléan considers invalid. Orléan considers invalid. Orléan considers invalid.
Given the aggregate nature of the \( \phi \), the marginal distribution of agent \( i \)’s aggregate excess demand for the asset at time \( t \) is therefore

\[
(i) \sum_{\phi=1}^{\phi} (i) \phi = (i)\mathcal{D}
\]
where \( \varphi \) is a factor measuring the liquidity of the market such that the average aggregate excess demand is zero (i.e., the market is not in mean-reversion). Nevertheless, in the hope that economic factors other than short-term excess demand engage in the evolution of the asset price, we focus here on the term excess demand may influence the evolution of the asset price, we focus here on the term excess demand may influence the evolution of the asset price.

We are concerned here with obtaining a result which could then be compared with actual market data and the short term excess demand is not an easily observable quantity. Also, most of the studies on the statistical properties of financial time series have been done on returns, log returns or price changes. We therefore need to relate the aggregate excess demand in a given period to the return or price change during that period. The excess demand needed to move the price by one unit is the excess demand needed to move the price by one unit. It is the excess demand needed to move the price by one unit.

\[
(\phi)_N \sum_{i=1}^{\infty} \frac{X_i}{N} = (\phi) x - (1 + \phi) x = x \nabla
\]

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to indicate that the price impact of trades may be non-linear. First, note that these studies deal with the price impact of trades and not of order flow. It is interesting to note that if $\Delta x = h(D)$, where $h$ is an increasing function of $D$

But the independent agent model is also capable of generating aggregate

impacts, empirical evidence tells us otherwise: the distributions

of dependent random effects is a plausible that a Gaussian description should

be a good one.

In order to evaluate the distribution of stock returns from Eq. (3), we need to know the joint distribution of the individual demands $\theta_i^t$. Let us begin by considering the simplest case where disturbances $\epsilon_i^t$ are independent and identically distributed random variables $\epsilon_i^t \sim N(0, \sigma^2)$. In this case the joint distribution of the individual demands is simply the product of the individual distributions and the price variation $\Delta x$ is a sum of $N$ iid random variables with finite variance. When the number of terms in Eq. (3) is large the central limit theorem applied to the sum in Eq. (3) tells us that $\Delta x$ is a mean of a large number of independent random variables, and the distribution of $\Delta x$ is a normal distribution. The need to know the joint distribution of the individual demands $\theta_i^t$ is shown in Eq. (2).

It is interesting to note that if $\Delta x = h(D)$, where $h$ is an increasing function of $D$, and if the individual demands $\theta_i^t$ are sequences of independent random variables (a somewhat extreme assumption), then it is easy to show that the overall wealth of all traders increases on average with time.
Individual demands is an essential character of the market structure and may not always be close to being a good approximation. The dependence between the "independent agent" approach, but also shows that such an approach is not always accurate. Indeed, if one relaxes the assumption that the individual demands have a finite variance, then under the hypothesis of independence (or weak dependence) of individual demands, the aggregate demand and therefore the price change (if we assume the model of linear demand, and therefore the price change is linear dependence of individual demands) will still be normally distributed. Therefore, the non-Gaussian and more generally non-stable character of empirical distributions, which are multimodal fermions of the notion of "weak" dependence, brings the use of dependent variables under various types of mixing conditions.

Mandelbrot's stable/Paretian hypothesis has been criticized for several reasons, one of them being that it predicts an infinite variance for stock returns. More precisely, a careful study of the tails of the distribution of increments for various financial assets shows that they have heavy tails with a finite variance. Many distributions verify these conditions; a particular example proposed by the authors and others is an exponentially truncated stable distribution. The tails of the density then have the asymptotic form of an exponentially truncated power law:

$$\frac{1}{\Gamma(\alpha)} \int_0^\infty \frac{1}{x} \exp \left(-\frac{x^\alpha}{\theta}\right) dx \sim (x^n)^{\alpha/\theta}$$

The exponent is found to be close to 1.5 for a wide range of stocks and market indexes. This critical point form allows for a heavy tails (excess kurtosis) without implying infinite variance. The central limit theorem also holds for certain sequences of dependent variables under various types of mixing conditions.

However, it is known the central limit theorem also holds for certain sequences of dependent variables under various types of mixing conditions. Therefore, the non-Gaussian and more generally non-stable character of empirical distributions, whether or not excess demand or the stock returns, not only demonstrates the failure of the "independent agent" approach, but also shows that such an approach is not always accurate. Indeed, if one relaxes the assumption that the individual demands have a finite variance, then under the hypothesis of independence (or weak dependence) of individual demands, the aggregate demand, and therefore the price change, will still be normally distributed. Therefore, the non-Gaussian and more generally non-stable character of empirical distributions, which are multimodal fermions of the notion of "weak" dependence, brings the use of dependent variables under various types of mixing conditions.
and do change the distribution of the resulting aggregate variable, indeed, the assumption that the outcomes of decisions of individual agents may be represented as independent random variables is highly unrealistic: such an assumption ignores an essential ingredient of market organization, namely the interaction and communication among agents.

In order to capture such effects we need to introduce an additional ingredient corresponding to the wealth of several investors that may be managed by a single fund manager. In a centralized market process, the wealth of the investors is managed by a single fund manager. In more general settings, the wealth of traders may be aligned through their decisions and act in concert. In the context of a financial market, several groups of traders may be aligned through their decisions and act in concert.

Indeed, the assumption that the outcomes of decisions of individual agents are independent and identically distributed is unlikely to emerge. In this sense of a mixing condition (1), we do not look at the aggregation procedure. They cannot be assumed to be independent and identically distributed.
presentation of the model

More precisely, let us suppose that agents group together in coalitions or clusters. Once a coalition has formed, all its members coordinate their individual demands so that all individuals in a given cluster have the same belief regarding future movements of the asset price. In the framework described in the preceding section, we will consider that all agents belonging to a given cluster will have the same demand for the stock.

More formally, let us suppose that agents group together in coalitions or clusters.

\[ \phi \sum_{\tau} \frac{W}{L} = \phi \sum_{\tau} X = \phi X \]

security analysts in the stock market, these coalitions may correspond to mutual funds, hedge funds, or even more specialized fund managers or to hedge funds in general that have the same demand for the stock. In the context of a given cluster, we may have the same demand for the stock. If a given cluster is formed, all its members coordinate their individual demands so that all individuals in a given cluster have the same belief regarding future movements of the asset price. In the framework described in the preceding section, we will consider that all agents belonging to a given cluster will have the same demand for the stock.

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where \( W \) is the size of the cluster and \( X \) is the (common) individual demand.
of the graph. Such an approach to communication in markets using random graphs was first suggested in the economics literature by Kirman [33]. Random graphs have also been used in the context of multilateral matching in search problems by Ioannides [30]. A good review of the application of random graph theory in the economics literature by Kirman [33].

The properties of large random graphs in the $N!/V^d$ limit were first studied by Erdős and Rényi [19]. An extensive review of mathematical results on random graphs is given in [8]. The main results of the combinatorial approach are given in Appendix 1. One can show that for $c = 1$ the distribution has an infinite variance if $d > 1$ and a finite variance if $d = 1$. The variance becomes finite because of the exponential tail.

For $c = 1$, the distribution has an infinite variance while for $c > 1$ the

$$\frac{z^2 M}{V} \sim M \int M d$$

(9)

is a power law.

The main results of the combinatorial approach are given in Appendix 1. One can show that for $c = 1$ the distribution has an infinite variance while for $c > 1$ the

$$\frac{z^2 M}{V} \sim M \int M d$$

is a power law.
In the limit $N \rightarrow 1$, the number of neighbors of a given agent is a Poisson random variable with parameter $\lambda$.
\[
\frac{\varepsilon(\varepsilon - 1)\varepsilon^N}{1 + \varepsilon} = (q)^N
\]

where \( \varepsilon \) is a function of \( c \) and the order flow \( q \). Substituting the above formula yields the kurtosis \( \kappa \) as a function of \( c \) and the order flow \( q \). Where \( N \) is the number of agents in the market (see Appendix A).

The moments may be obtained by an expansion

\[
\frac{(x^\varepsilon)^r}{(x^\varepsilon)^r} = (q)^N
\]

An interesting quantity is the kurtosis of the excess demand, which in our model is equal to the kurtosis of the asset returns with:

\[
\frac{(x^\varepsilon)^r}{(x^\varepsilon)^r} = (q)^N
\]

The variance of excess demand is calculated through a Taylor expansion of Eq. (1) (see Appendix B for details). The calculation of the moments of \( x^\varepsilon \) may be obtained by an expansion in \( 1/N \) where \( N \) is the number of agents in the market (see Appendix C).

Substituting their expression on the above formula yields the kurtosis of excess demand \( \kappa \) as a function of \( c \) and the order flow \( q \). The moments may be obtained through a convolution product, and being the number of clusters, where \( \otimes \) denotes a convolution product, and \( x \) being the number of clusters,

\[
(x^\varepsilon)_{\otimes} = \sum_{q} \left( \frac{1}{\varepsilon} \right)^{\varepsilon} \int (x^\varepsilon)_{\otimes} (y^\varepsilon) d \frac{1}{\varepsilon} = (x = x^\varepsilon)
\]

The distribution of the price variation is then given by

\[
\sum_{q} \left( \frac{1}{\varepsilon} \right)^{\varepsilon} \int (x^\varepsilon)_{\otimes} (y^\varepsilon) d \frac{1}{\varepsilon} = (x = x^\varepsilon)
\]

such as NYSE, NASDAQ, etc., for a period of minutes, or a liquid market, of different orders corresponding to different demands, as defined above, of different order sizes. The number of orders received during the time period \( t \) is assumed to be random, and the number of market participants who actively trade in the market during a given period, for example, \( a_{\text{market}} \) can be thought of as \( a_{\text{market}} \).
where \( A / (c /) \) is a normalization constant with a value close to 1 defined in Appendix 1, tending to a finite limit as \( c \rightarrow 1 \). This relation may be interpreted as follows: a reduction in the volume of the order now results in larger price fluctuations, characterized by a larger excess kurtosis. This result corresponds to the well-known fact that large price fluctuations, characterized by large excess kurtosis, tend to occur in less active markets, characterized by a smaller order volume. Hence, we have exhibited a model of a stock market which, albeit its simplicity, gives rise to non-normal probability distributions for aggregate excess demand and price fluctuations, characterized by a larger excess kurtosis.

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### 2 Discussion

It is also consistent with results from various market microstructure models. If the market maker, herding tendency tends to be stronger during time-dependent, for example, herding tendency tends to be stronger during time-dependent. For example, herding tendency tends to be stronger during time-dependent. Therefore, the asymptotic behavior of \( P(x) \) is still of the form given above, i.e., \( \text{Order} \approx \sqrt{N} \). However, even after accounting for heteroskedasticity, the conditional distribution of stock returns for small firms is flatter than that of large firms, small firm

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\[ \text{Order} \approx \sqrt{N} \]
model illustrates the fact that while a naive market model in which agents do not interact with each other would lead to giving rise to normally distributed market returns, the model incl.
effect of prices on the behavior of market participants: a nonlinear coupling between can lead to a control mechanism maintaining criticality.

Yet another interesting dynamical specification compatible with our model is obtained by considering agents with "threshold response". Threshold models have been previously considered as possible origins for collective phenomena in economic systems. One can introduce heterogeneity by allowing the individual thresholds to be random variables. For example, one may assume the thresholds to be independent random variables with a standard distribution. Threshold-modified price variations in a stock market with many agents can be simulated by adjusting the parameters in a model that captures the dynamics of price fluctuations.

References


Appendix I: Some results from random graph theory

Unless specified otherwise, $N, \sigma$ means $N, \sigma \in \mathbb{N}$. 

\[
\sigma \leq N \implies \frac{\binom{\sigma}{N}}{\binom{\sigma}{\sigma}} \leq 1
\]

Appendices


Appendix 1: Some results from random graph theory

-uniformly in $c$ on all compact subsets of $[0,1]$

\[
\lim_{N \to \infty} \frac{\binom{\sigma}{N}}{\binom{\sigma}{\sigma}} = 1
\]

Unless specified otherwise, means $\binom{\sigma}{N} \sim (\sigma/N)^c$.

Appendices


In this appendix we will review some results on asymptotic properties of large random graphs. Proofs for most of the results may be found in [1] or [8].

Consider $N$ labeled points $V_1, \ldots, V_N$, called vertices. A link (or edge) is defined as an unordered pair of vertices $\{V_i, V_j\}$, where $i \neq j$. A vertex is linked to itself by an edge. A link is called a free edge if it is connected and if none of its subgraphs is a cycle. A graph is called a free graph if it is connected and if none of its subgraphs is a cycle. A vertex is linked by a path to another vertex if there exists a path that begins at the vertex and ends at the other vertex. A graph is said to be connected if any two vertices are linked by a path. A graph is called a tree if it is connected and if none of its subgraphs is a cycle.

In the following, we will be interested in the properties of large random graphs whose distribution only depends on $N$ and $c$. We shall be particularly interested in the case $p = c/N$. A graph of type $(\mathbb{R}, N)$ is a random graph obtained by a random process such that each edge is formed independently with probability $p$. The decision for different edges being included or not is made by choosing, for each pair of vertices $V_i, V_j$, whether to link them or not through a random process, the probability for which is $p > 0$. The distribution of the graph obtained by such a process is termed a random graph distribution.

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which gives in the large limit
\[ N\left[ \frac{N}{\Phi} \right] = (1 + \frac{N}{\Phi})^{N} \]

Multiplying both sides by \( e^{z} \) and summing over \( s \) gives:
\[ \left( e^{z} \right)^{N_{d}} \prod_{s=1}^{N_{d}} \left( e^{z} \right)^{N_{d}(s-1 + \cdots + s + 1)} = \left( e^{z} \right)^{N_{d}} \]

The above expressions are valid for \( \Phi \neq 1 \).

\[ \sum_{s=1}^{N_{d}} \left( \begin{array}{c} N_{d} \\ s \end{array} \right) = (s)^{1+N_{d}} \]

The abovementioned distribution of cluster sizes in a large random graph is asymptotic to the moment generating function of the distribution.
The $j$-th moment of $u^2$ is then given by:

$$z \varphi(\gamma = u^2) R d \sum_{N} = \varphi(u^2) = (\varphi^2 \gamma) N \varphi$$

where $\varphi(u^2)$ is to be defined via an approximation of $\varphi$.

Define the moment generating function $\varphi(u^2)$ for the var-

We shall relate this result below, and proceed to calculate higher moments

$$(1) \varphi + (\frac{1}{\gamma} - 1) N = (\varphi^2 \gamma)$$

which implies that

where $\lambda(N)$ is the number of independent cycles and $\xi(N)$ the number of

$$(\varphi^2 \gamma) = (\varphi^2 \gamma) N + N - (\varphi^2 \gamma)$$

From a well known generalization of Euler's theorem in graph theory

$$\frac{\gamma}{\varphi^2 \gamma} \sim N \xi$$

The $j$-th cumulant of $u^2$ is then proven by:

An asymptotic normal distribution when $\lambda(N) \to \infty$ and that for large $N$ the

and the parameter $c$ in the section we will show that $\xi(N)$ depends on $\lambda(N)$ and the parameter $c$ in this section we will show that $\xi(N)$

Let $N$ be the number of clusters (connected components) in a random

Appendix 3: Number of clusters in a large random graph

where $\sigma(s)$ is a normalizing constant defined such that $s = 1$.

$$s \frac{1}{d} \sigma = 1$$

$\lambda(\varphi)$ is then proven by:

The distribution of clusters sizes $(d) \sigma + \varphi = (\varphi^2 \gamma) \sigma + \varphi$ from which various moments and cumulants may be calculated recur-

$$(1 - (\varphi^2 \gamma) \varphi + \varphi = (\varphi^2 \gamma) \varphi + \varphi$$
Let us also consider the cumulant generating function \( \Phi(z) \) defined by

\[
\Phi(z) := \exp(\sum \frac{C_j}{j!} z^j)
\]

The \( j \)-th cumulant of the distribution of \( N \) may then be calculated

\[
C_j = \frac{\partial^j}{\partial z^j} \Phi(z) \bigg|_{z=0}
\]

We will now establish an approximate recursion relation between \( N \) and \( N+1 \). Take a random graph of size \( N \), the probability of a link between any two vertices being \( p = \frac{c}{N} \). In order to obtain a graph with \( N+1 \) vertices, note that the addition of a new vertex and choose randomly the links between the new vertex and the other \( N \) vertices. In two vertices being \( p = \frac{c}{N+1} \). In order to obtain a graph with \( N+1 \) vertices, take a random graph of size \( N \), the probability of a link between any two vertices being \( p = \frac{c}{N} \). We will now establish an approximate recursion relation between and

\[
\Phi(z) = (c, N) \Phi(z)
\]

Let us also consider the cumulant generating function defined by \( \Phi(z) \) as the logarithm of the distribution of \( z \) then be calculated

\[
\exp \frac{\partial}{\partial \Phi} \Phi(z) = (z, \Phi(z))
\]
Nevertheless for the large-$N$ limit, $Z - N/2$ is a Poisson variable with parameter $N/2$. Without rescaling, we have

$$\frac{1}{N} \mathbb{E} \left[ \frac{1}{N} - \frac{Z - N}{N} \right] = (\eta = Z) \text{d}$$

Note that the asymptotic forms of cumulants of $\eta$ are identical to those of a random variable with the following distribution:

$$\mathbb{E} \left[ \eta \right] \sim N \varepsilon$$

Substituting $(\eta = Z)$ into $(\eta = N) = N \varepsilon$ yields a simple differential equation for the cumulants in the large-$N$ limit with respect to $\varepsilon$:

$$(1)O + N(\varepsilon)^{\mu} = (\varepsilon N)^{\mu}$$

Let us now derive a similar relation for the variance $\mu$ of $Z$ defined as the variance of $\eta$ with respect to $\varepsilon$:

$$(1)O + N \left( \frac{Z}{\varepsilon} - 1 \right) = \frac{z}{\text{d}}$$

whose solution is

$$\varepsilon - 1 = (\varepsilon)^{\mu}$$

Substituting in $(\mu = Z)$ yields a simple differential equation for the cumulants:

$$(1)O + N(\varepsilon)^{\mu} = (\varepsilon N)^{\mu}$$

Let us first retrieve the result given in Appendix A for $u$. Define $u$ such that $\varepsilon = u / \sqrt{N}$ and $N$-graph theory can be used to translate the fact that the probability for a link has to be renormalized when going from a $N$-graph to a $N+1$-graph.

By taking successive partial derivatives of $(\varepsilon = Z)|_{Z} = \rho$ with respect to $z$ and renormalizing, we obtain:

$$\frac{1}{N} \mathbb{E} \left[ \frac{1}{N} - \frac{Z - N}{N} \right] = (\eta = Z) \text{d}$$
Both $N$ and $\Phi$ are analytic functions of $z$ in a neighborhood of finite $N$.

Let $Y = N^c / (1 - c)$, then

$$Y = N^c / (1 - c) = 2qN$$

where $q$ is the number of clusters or trading group, i.e., the number of

$$X \sum_{x \in N} ^{1} = \phi \mu \sum_{x \in N} ^{1} = x \nu$$

The relation between $X$ and other variables of the model is given by equation: $X = X_0 / X_0$. In this appendix we derive an equation for the generating function of the variable $x$ which represents one-period return of the asset.

$$\sum_{n=0}^{\infty} \left( \frac{x}{x} \right) = \frac{N^c / (1 - c)}{N^c / (1 - c) - z}$$

We show that the convergence of the cumulants implies convergence in distribution. Under the conditions, one can show that the convergence of the cumulants implies convergence in distribution. The only distribution with zero mean, zero variance, and zero higher cumulants has zero mean and unit variance and its higher cumulants tend to zero:

$$N = \frac{z^2 / \nu N / (\nu - 1) N - z}{N} = N^c / (1 - c)$$

or, consider now the rescaled variable:

$$N^c / (1 - c) = N$$

while both $N$ and $\Phi$ are analytic functions of $z$ in a neighborhood of
In order to calculate this sum, let us introduce the moment generating functions for \( x \) and \( X \):

\[
\left( \frac{N}{1} \right)^\phi + \left( \frac{N}{1} \right)^\phi \left( 1 - \frac{N}{1} \right)^{x_{\text{exp}}N} = (1 - \frac{\phi}{1})
\]

\[
\left( \frac{N}{1} \right)^0 + \int \left( \frac{N}{1} \right)^{x_{\text{exp}}N} dx = N^\phi
\]

We denote by \( x_{\text{exp}}N \) the number of orders that in the limit \( N \rightarrow \infty \) are active in the market in one period. \( x_{\text{exp}}N \) is the average number of buy orders in a given period. Therefore \( x_{\text{exp}}N \) is the number of clusters defined in appendix 3. \( x_{\text{exp}} \) is an analytic function whose series expansion is given by the cumulants of \( x_{\text{exp}} \) defined in appendix 3, \( \Phi \) is the cumulant generating function of the number of clusters:

\[
\int \left( \frac{N}{1} \right)^{x_{\text{exp}}N} dx = \left( \frac{N}{1} \right)^{x_{\text{exp}}N}
\]

One can evaluate the above sum in the large \( N \) limit as

\[
(1 - \frac{\phi}{1})\left( \frac{N}{1} \right)^{x_{\text{exp}}N} = \frac{\int x_{\text{exp}}}{\phi x_{\text{exp}}} \sum_{n=1}^{\infty} \frac{\phi}{n} N + z N = (z)\Phi
\]

where \( \Phi \) is the cumulant generating function of the number of clusters.

\[
\int \left( \frac{N}{1} \right)^{x_{\text{exp}}N} dx = \left( \frac{N}{1} \right)^{x_{\text{exp}}N}
\]

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\int \left( \frac{N}{1} \right)^{x_{\text{exp}}N} dx = \left( \frac{N}{1} \right)^{x_{\text{exp}}N}
\]

\[
\int \left( \frac{N}{1} \right)^{x_{\text{exp}}N} dx = \left( \frac{N}{1} \right)^{x_{\text{exp}}N}
\]

Multiplying the right hand side of the equation above by \( s \) and summing over \( x_{\text{exp}} \) yields

\[
z_{\text{exp}}(s) = x_{\text{exp}} \Phi d \sum_{n=1}^{\infty} \frac{\phi}{n} N + z N = (z)\Phi
\]

In order to calculate this sum, let us introduce the moment generating function for \( x_{\text{exp}} \) and \( x_{\text{exp}} \):
\[
\frac{(vX)^{1/2}(-1)^{v,\alpha,\varepsilon}N}{(vX)^{1/2}} = (A)^{1/2}
\]

which implies that the kurtosis of the aggregate excess demand is

\[
\frac{(vX)^{1/2}(-1)^{\alpha,\alpha,\varepsilon}N}{(vX)^{1/2}} + (vX)^{1/2}(-1)^{\alpha,\alpha,\varepsilon}N = (A)^{1/2}
\]

\[
(vX)^{1/2}(-1)^{\alpha,\alpha,\varepsilon}N = (A)^{1/2}
\]

Let us now examine the implication of the above relation for the moments.

One finally obtains:

\[
\frac{(N)}{1} + [(1 - (z)f)(\frac{\varepsilon}{\gamma} - 1)^{\alpha,\alpha,\varepsilon}] =
\]

\[
[(1 - (z)f)^{\alpha,\alpha,\varepsilon}] = (z)^{1/2}
\]

in the above expression