Price Variations in a Stock Market with Many Agents

They define

of rational traders is larger, the market price is generally locked within the price range moves outside the range justified by fundamental market analysis. When the number of rational traders is small, "bubbles" often occur, where the market price governed solely by studying the market dynamics is investigated. When the relative analytical of the stock, including dividends, and "noise traders", whose behavior is induced by each other's behavior, is explored between "rational" traders whose behavior is derived from fundamental in the simplest, least realistic case, exact results for the statistics of the variations are derived by mapping onto a model of diffusing and annihilating particles, which has been solved by quantum field theory methods. When the agents imitate each other, different scaling behavior is obtained. In this case the statistics of price variations is consistent with empirical observations. The Levy stable distribution proposed by Mandelbrot to describe real market indices, the low stable distribution proposed by Alabdulrhman to describe real market indices, over different time scales can be related to each other in a systematic way, similar to the interplay between "rational" traders whose behavior is derived from fundamental analysis of the stock, including dividends, and "noise traders", whose behavior is governed solely by studying the market dynamics. The variations constituting simple models of a stock market, and argue that the large variations may about existing models, which all fail to account for non-Gaussian statistics, We large variations in stock prices happen with sufficient frequency to raise doubts.
I. INTRODUCTION

The literature on competition among firms and the literature on stock market prices has been for the most part treated separately. Here we adhere to this approach, although even-though for the most part treated separately, the literature on competition among firms and the literature on stock market prices has

ranges from a minute to a day, with convergence to a Gaussian at approximations on a time scale which is reasonably shifts by a truncated Levy distribution with a ~ L(t) over a time scale which Gaussian distribution at long time scales. For example, the data for the S&P 500 index distribution formally account for slow convergence to a probability distribution of an intermediate scaling regime, where price changes occur with a probability distribution progressively converges to a Gaussian (Ackerly and Poon, 1988). The presence finite. Empirical observations suggest that the succession of daily, weekly, and monthly distributions where the scaling regime for price fluctuations is actually finite rather than truncated Levy distribution where large events (beyond ~ a) are probability unlikely.

The Levy distribution has substantially more weight for large events than the Gaussian, where integrals could be described by a stable Levy distribution, rather than being Gaussian, observed that price variations of many market indices over different, but relatively short existing models, which all fail to account for non-Gaussian variations in price. Mandelbrot observed that price variations, which all fail to account for non-Gaussian variations in price, have been happening with sufficient frequency to raise doubts about existing models (1966, 1966) on the nature of stock prices and the recent behavior of the derivative markets A modification of this work is suggested by the early observations of Mandelbrot (1963), price variations in a stock market.

minimal structure needed to make an adequate description of the statistical properties of economic models tends to become economically complicated when attempts at realistic economic models must be linked to the paper economy of finance. Even the simplest model which makes explicit the feedbacks between the "real" or physical economy and the financial markets must be linked to the paper economy of finance.
Our results suggest that large fluctuations in price may be endogenous to the dynamics of
markets. Our results are a collective effect resulting from many interacting agents.
Variables in our model are a convolution of these variables. In addition, we are assuming
price forms a dynamic process that is sufficiently robust to describe price variations in real
markets. One version of the model exhibits power law fluctuations at small time scales
in spite of these gross simplifications, some of our toy models exhibit a statistical pattern

\[
\begin{align*}
\text{Hurst exponent } H & \approx 1 / 2, \\
\text{Fat tails } & \sim H \approx 1 / 2.
\end{align*}
\]

\text{We construct an extremely simple, but completely defined, economic model of many}

\text{agents each optimizing their own utility functions. There is only one type of stock and each}

\text{agent can own at most one share. These simplifications are quite drastic for many reasons.}

\text{First of all, in reality the price changes of different stocks are correlated to each other. Thus}

\text{one cannot treat each stock as an independent market. In addition, we are ignoring price}

\text{changes due to exogenous changes such as interest rates, money supply, or wars breaking}

\text{out. Finally, we have an extremely simple description of the individual agent's behavior.}

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A. Review of Previous Literature

Zipf (1949) wrote a provocative book in which he observed regularity in the distribution of most frequently used words in English and several other languages. He also suggested that in probabilities of future market share, positive feedback leads to "history dependence" relative sizes of current market share and offered a policy process for updating the change to scale. He considered the probability of future market share to be dependent on the size of market share based on what might be called "network intersection returns" as suggested by Gilburt. More recently, Arthur (1989a, 1990) looked at production lines where the size distribution of firms where the driving mechanism is constant returns at any size, as described above. These observations have thus far compounded all attempts at a general explanation. The truncated Pareto-Levy distribution has been used to describe various economies of scale, the truncated Levey distribution has been used to describe various economies of scale, characterized by a Hurst exponent larger than that for a random walk ($\zeta/1 = H$). A related observation was that the temporal correlations in price fluctuations were characterized by a Hurst exponent $H \approx 0.6$, larger than that for a random walk. A related observation was that the temporal correlations in price fluctuations were characterized by a Hurst exponent $H \approx 0.6$, larger than that for a random walk.

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Based on an analogy with traffic flow, Paczuski and Nagel (1996) have speculated that a critical state with fluctuations of all sizes might be the most efficient state that can be achieved in an economy. Criticality (Bak, Tang, and Wiesenfeld, 1987, 1988) is based on an analogy with traffic flow,
of mass markets with diverse agents who may initiate influence on fluctuations in stock prices. Key to our approach is to consider the dynamics of market price variations, and to examine their scaling behavior. In our models we attempt to isolate several of these sources and to examine their effective Hurst exponent. An apparent scaling regime exists for the model.
Heuristically we offer the following sketch of the extremely simple economy we model.

A. The Firm and Stock

Heuristically we offer the following sketch of the extremely simple economy we model.

B. The Agents

This does not affect the agents' behavior. The agents' decisions are affected by the random variations of dividend payments, but we assume that the decision in the long run where dividends are paid many times. Of course, the liquidity of the dividend distribution affects the dynamics of the economy, and hence our evaluation of the agents' performances. This regularization does not affect our evaluation of the agents' profits and losses of the traders, we assume that at each instant every agent who owns stock will buy stock at the dividend distribution function to be defined below. However, it is still true to do the bookkeeping for the dividend distribution affects the dynamics we are interested in only through the

For simplicity we consider with probability \( d \) and without probability \( (1 - d) \), for simplicity we consider with probability \( p \) and without probability \( (1 - p) \), for instance, determined by a Bernoulli process. The dividend is a random variable, for instance, determined by a Bernoulli process. The dividend is a random variable, for instance, determined by a Bernoulli process.

In our model, the presence of a single firm is only implied in a lottery ticket or shares of
by period utility function which might be of the form

We assume that there is another type of trader who maximizes his short run or period

1. An Optimizing Agent with a Utility Function

score we may do so.

the (in general, unobservable) utility function for each agent. Although we wish to "keep"
not postulating an explicitly optimizing behavior for these agents we do not need to specify
own a share is a potential buyer. It is not allowed to own more than one share. As we are
Each individual who owns a share is a potential seller and each individual who does not
individual of type 1 are (1, W) if he owns stock, and (0, W) if he owns no stock.
the population are potential buyers and half potential sellers. The initial endowments of
of this type and N - M traders of the next type. There are N/2 shares available: hence half of
all could profit if they did not overpay for a stock. We assume that there are N - M traders
make money. The presence of dividends converts the process into a nonzero sum game and
at poor prices, but if the dividend rate is sufficiently relative to the price he pays he could
Exactly type 1 individual is a simplistic trader who might lose money if he chirns too much
converging to the Gaussian result.

grow in time faster than a random walk over a broad range of time scales, before eventually
in the unmechanism we are positing gives rise to anomalous large fluctuations that
context by Maninna and Paczuski (1996). Here we find that the positive feedback inherent
beaked structure of oragnized agents leading to avalanches of all sizes is found in a different
a beaked and path dependent as noted by Arthun (1990). The win process also can create a
by (sell). In general this type of "buy" or "sell" process is responsible for positive
the new buyer (seller) picks another buyer (seller) at random and copies the same price
that traded removed. The new price is either chosen randomly within a finite interval or
are many think of this as a new buyer and seller entering the market, with the old ones
The current seller (buyer) now becomes a buyer (seller) at a new price. Equivalently,
The interpretation of this equation is that each individual has an overall risk-averse utility function which is a mix of the extremes of risk aversion and risk neutrality.

\[
(1) \quad (G^2 - 1) + v d (a - 1) + \min \{G, v \} \text{ with } a = \eta
\]
expectations are formed out of equilibrium. Is not already there. No learning or other behavioral theory is supplied to indicate how
based on their acceptance. Nothing is said about convergence to equilibrium. If the system
expectations concerning prices the expectations will be self validated by optimizing behavior
the problem of the formation of expectations by the argument that given the appropriate
more than another way of stating that a noncooperative equilibrium exists. If these steps
for a complete passage of all dynamics, the magic invocation of rational expectations is nothing
If one considers only equilibrium with consistent expectations it is possible to engage in
acceptance of a weak form of local optimization defined at each fixed point in the system.
acceptance of a weak form of local optimization defined as a fixed point in the system.
the proposition that all individuals must have mutually consistent expectations implies the
expectations are handled cannot be avoided. Considering the individual's optimization on
is considered it is consistent on expectations of market price. A specification of how ex-
...
In essence, a game theoretic solution to an n-person game in extensive form is nothing more than a set of strategies which completely describe the motion of the system, the size of which may be highly sensitive to information conditions.

B. Information Conditions and Expertise

Optimizers and a "mass particle behavioral approach" of the others, who consider both the game theoretic solution of noncooperative equilibria and the dividend condition where no guarantee is made as to contributions will be made, will be found to be inappropriate as the rational conditions require that in equilibrium both rational expectations and the rational expectations assumption of being rational will be made. However, these conditions may be interpreted as the rational expectations conditions requiring that if an equilibrium exists, all individuals must be consistent with expectations, all consistent with expectations can invoke a consistency of expectations. A game is played forward sequentially and cannot avoid dealing with the equilibrium condition on rational expectations.

A. Expectations

In essence, a game theoretic solution to an n-person game in extensive form is nothing more than a set of strategies which completely describe the motion of the system, the size of which may be highly sensitive to information conditions.
C. Loans and Bankruptcy

If it were possible for the individuals to borrow each period then we would have to take


D. Equilibrium and Behavioral Finance


E. Equilibrium and Behavioral Finance


F. Equilibrium and Behavioral Finance


G. Equilibrium and Behavioral Finance


H. Equilibrium and Behavioral Finance


I. Equilibrium and Behavioral Finance


J. Equilibrium and Behavioral Finance


K. Equilibrium and Behavioral Finance


L. Equilibrium and Behavioral Finance


M. Equilibrium and Behavioral Finance


N. Equilibrium and Behavioral Finance


O. Equilibrium and Behavioral Finance
the noncooperative equilibria and contrast them with the behavioral models.

E. A Few Crude Facts

In order to sweeten our intuition for this type of model, a few statistics selected from the New York Stock Exchange (NYSE) supply some orders of magnitude. If we consider that the period of time for which an individual owns a portfolio of stock is somewhere between 20-50 years, then the pure OLG contribution to stock trading for a constant population is somewhere between 2.5% on the conservative assumption that stock will be sold rather than re-purchased on the New York Stock Exchange (NYSE) in 1994. If we consider that the noncooperative equilibrium will involve all individuals holding an optimal portfolio of stocks and never trading, what are the factors that account for stock market trading and how expectations equilibrium will involve all individuals holding an optimal portfolio of stocks for the period of time for which they own a portfolio of stock is somewhere between 20-50 years, we observe that in the actual New York Stock Exchange (NYSE) in 1994, the volume of stock traded was 34%. 

In 1994, approximately 72 million public offerings raised $82 billion, institutional investors accounted for $52 billion, giving an average value of around $71 per share and an average issue of around 2.6 million. In 1993, there were around 1.0 million institutional investors in the United States, and around one-third of the market value of all stocks was held in the New York Stock Exchange (NYSE). In 1994, a few crude facts

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Karatzas, Shubik, and Sudderth (1995) and Due, Geanakoplos, Mas-Colell, and McNel-
If the randomly chosen agent did not own a share and there are sellers willing to sell at a

may or may not be related to his observations of the market. The maximization assumption is essentially normative and although it might hold in circumstances where individuals are experts and dedicated money maximizers it does not necessarily provide a good empirical view of actual behavior. If there is a substantial segment of the market following some other rule the presence could influence the environment in which the market operates. Before discussing the various models which have been studied numerically, we first define

\section{Simulations of the Market}

The formal optimizers act. Our investigations in Section IV consider this possibility.
A Market with Fundamental Value Buyers Only.
all the numerical simulations to follow, we shall assume that the rational agents had already reached an equilibrium state with no trade at the beginning of the simulation. 

B. A Market with Simple "Noise Traders" Only.

At the other extreme, let us consider a market where all the agents trade according to their observations of the state of the market without concern for the fundamental values. In principle, their behavior can be defined in a number of different ways. For simplicity, we may initiate the simulation in a state where the $N/2$ agents who own stocks are willing to sell at prices which are uniformly distributed in the interval $p_{	ext{max}}/2 < p < p_{	ext{max}}$. We may initiate the simulation in a state where all the agents who do not own stocks are uniformly distributed in the interval $0 < p < p_{	ext{max}}/2$. For each update step, the price $p(t)$ is chosen randomly, with equal probability in either the downward or upward direction. Most of the time this will not induce any trade, but occasionally the chosen agent will find himself at a price level where other buyers or sellers become interested in a trade. How does the price $p(t)$ vary with time?

I. Independent Noise Traders

More complex behavioral models with a higher degree of interaction between agents, when there is an overlap in prices. We consider the simple model first, before investigating other selling or buying prices in the market. The agents interact only by buying and selling. The simplest behavioral model is when each agent's price fluctuates randomly, independent of the other agents. Perhaps the simplest way to define the behavior of the noise traders is to choose an asking price randomly between $p$ and $p_{	ext{max}}$ if the chosen agent is a buyer, or between $p_{	ext{min}}$ and $p_{	ext{max}}/2$ if the chosen agent is a seller. Then, the new bidding price will be taken at a price $p(t)$, and the new buyer will choose a new bidding price for possible future trades. The new seller will choose a new midpoint price for possible future trades. After the sale, the new buyer will choose a new midpoint price for possible future trades.
It turns out that a very similar process has been studied extensively by physicists, and there is no type of scaling that can occur in systems with a few degrees of freedom, or agents in our case. The formalism used to derive Eq. (4) makes use of Quantum Field Theory, and there is no guarantee of the interactions between very many agents in a simple market model. This correspondence of the interactions between very many agents in a simple market model, can arise as an accidental result of the model. Nevertheless, it demonstrates rigorously that the global confinement of the price range. We stress that the situation considered here is pretty academic in the context of economic theory. Nevertheless, it demonstrates rigorously that anomalous scaling behavior, like the one observed for real markets, can arise as a consequence of the interactions between very many agents in a simple market model. Equation (4) has been shown mathematically that the variation of the price after time \( t \) scales as

\[
\frac{\Delta p(t)}{p_0} \sim \frac{1}{t^{1/4}}
\]

where the distribution after a long time is a Gaussian with a width scaling as

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and has been extensively studied.

Barkema, Howard, and Cardy (1996) have shown mathematically that the variation corresponds to the market price \( p(t) \), the reaction-diffusion process is called “A+X→B”, \( A \) and \( B \) are represented by the price \( p \) and \( t \), respectively. The position where the annihilation event takes place is at the opposite ends of the tube (Figure 1). The positions of the two types of particles correspond to the particle random walks. When the two types of particles are injected at the opposite ends of the tube, the variation of the price after time \( t \) scales as

\[
\Delta p(t) \sim \frac{1}{t^{1/4}}
\]

This result holds for very long time scales, the price variations are scaling with

\[
\frac{\Delta p(t)}{p_0} \sim \frac{1}{t^{1/4}}
\]

where the parameter \( t_0 \) could depend on the number of particles and the size of the price range.

For shorter time scales the variations are larger because of the logarithmic factor in Eq. (4). For the longest time scales the variations are scaling with

\[
\frac{\Delta p(t)}{p_0} \sim \frac{1}{t^{1/4}}
\]

where the distribution after a long time is a Gaussian with a width scaling as

\[
\frac{\Delta p(t)}{p_0} \sim \frac{1}{t^{1/4}}
\]

and has been extensively studied.
The behavior of the Independent Noise Traders model is illustrated in Figure 2(a,b,c,d). The system includes \(N = 500\) agents operating within a price range of \(p_{\text{max}} = 500\). Figure 2a shows the variation of the price vs time. Figure 2b shows the histogram for the distribution of agents' prices at some instant in time. The 200 agents to the right of the gap are sellers, while the 200 agents to the left of the gap are buyers. The number of potential buyers or sellers is relatively small near the gap, which defines the current market price. Agents differing into this regime will trade, or "annihilate," and be shifted to other values on the other side of the gap. A result of the logarithmic term, these are both rigorous results.

The price variations in the Independent Noise Traders model asymptotically have a power-law form described by Mandelbrot, although the exponent is smaller rather than larger than that of a Gaussian distribution. The price variations can be conventionally represented by a Hurst plot (Feder, 1989). The price variations in the Independent Noise Traders model asymptotically have an exponent smaller rather than larger than that of a Gaussian distribution. However, at short time scales the model exhibits larger than Gaussian fluctuations as short time fluctuations one might otherwise consider that the exponent is much greater.

Thus, for small the effective exponent for the fluctuations in Figure 2c is much larger than scales which occur in the slowly varying long time power law behavior, as seen in Figure 2a. The logarithmic factor represents the large noise or glitches at shorter time correlations. The logarithmic factor gives rise to slower convergence to the asymptotic value. From the theoretical result for the equivalent A/B process, we predict \(\log R(t)/\log t = 1/4\), and for smaller \(t\), the apparent slope is larger because of the logarithmic process, we predict \(H = 1/4 + \log(\log t)/\log t\). Hence, for large \(t\), the slope approaches \(0 < \log R(t)/\log t < 1/4\) for the simulation. From the theoretical result for the equivalent A/B process, we predict \(H = 1/4 + \log(\log t)/\log t\). Hence, for large \(t\), the slope approaches \(0 < \log R(t)/\log t < 1/4\) for the simulation. From the theoretical result for the equivalent A/B process, we predict \(H = 1/4 + \log(\log t)/\log t\). Hence, for large \(t\), the slope approaches \(0 < \log R(t)/\log t < 1/4\) for the simulation.
One might naively have suspected that the model would exhibit pure random walk behavior for the price variations, since it is based on the diffusive behavior of the individual agents. After a trade, the current buyer and seller would choose a new price randomly. A more intricate situation arises if the agents, when choosing their asking prices and bids, instead of choosing between zero and infinity, choose prices from a box of size $p_{max}^*$ These noise traders at least realize that prices have to be within a certain range - otherwise they would have no clue to choose their prices between zero and infinity.

The slow price variations are related to the fact that the prices are controlled by past to a logarithmic factor responsible for slow convergence to the asymptotic result. There is excellent agreement between our numerical results and the previous analytical work on the reaction-diffusion system, although the price variations exhibit a drift coefficient $D = 0.5$ was chosen. Figures 3, 4, and 5 correspond to a drift $D = 0.5$, and one step away from the market price with probability $1/2$, and one step towards the market price with probability $1/2$.

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After a trade, the current buyer and seller would choose a new price randomly. A more interesting situation arises if the agents, when choosing their asking prices and bids, mimic existing traders in the market. This type of drift behavior is related to the fact that the prices are confined to a box of size $p_{max}^*$. These noise traders at least realize that prices have to be within a certain range - otherwise they would have no clue to choose their prices between zero and infinity.

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The Urn Model

We will consider a process, where after each transaction the new selling and buying prices for the trading agents are chosen by randomly picking a buyer and seller and copying his price. This has the effect that the new price is chosen with a probability proportional to the number of traders who currently have that price. However, no global information on the part of buyers and sellers is needed when choosing a new price. This opens up the possibility of mimicking crowd behavior, where agents follow each other's actions without paying any respect to fundamental market values.

C. The Urn Model

Long time scales:

\[ H = 1 \]

However, no global information on the part of buyers and sellers is needed when choosing a new price. This opens up the possibility of mimicking crowd behavior, where agents follow each other's actions without paying any respect to fundamental market values.

\[ p_{\text{max}} = 4000 \]

Although it could be made arbitrarily large. The collection of agents organize themselves into a well-defined price range (Figure 4b), which is not affected by the external parameter \( p_{\text{max}} \). As long as \( p_{\text{max}} \) is sufficiently large, this is in contrast to the case of independent noise traders (see Figures 2b, 3b) where all up the entire interval \([0, p_{\text{max}}]\) is filled up with agents, which is not accessible by the external parameter \( p_{\text{max}} \) as long as \( p_{\text{max}} \) is sufficiently large. The collection of agents organize themselves into a well-defined price range (Figure 4b) which is not affected by the external parameter \( p_{\text{max}} \) as long as \( p_{\text{max}} \) is sufficiently large. The collection of agents organize themselves into a well-defined price range (Figure 4b) which is not affected by the external parameter \( p_{\text{max}} \) as long as \( p_{\text{max}} \) is sufficiently large.

\[ p_{\text{max}} = 4000 \]

Although it could be made arbitrarily large.
Again, though the convergence is very slow, although we presently have no analytical economic reasons why large fluctuations create a need for positive feedback in the change in the market price, this observatory is a measurement for positive feedback in the change of moving one unit up or down, the agents move d units. Since as before, thus instead of the probability for increase is $2/(1 + 1/2)$ and decrease is $2/(1 - 1/2)$, in a small update his price will increase or decrease his price randomly by an amount $\Delta p$

In the simulation, if the price change during the last period of 100 time units is

\[
\Delta p
\]

the actual recent variations of market prices.

dealt feature that the diffusion constant and drift for the agents’ price is proportional to variations in the price variance. To mimic this effect, we simulated the urn model above with the actual recent changes in the market price. Although it could go either up or down, we thereby that there will be a large variance the next day, a large drop on a given day; it is likely that these will be a large variance. It has recently been observed that various market indices exhibit volatility clustering. At times when prices have recently been volatil, this volatility could influence the behavior of the noise traders, or the imitators. If the Dow-Jones index exhibits volatility clustering, then on days when the index has gone up or down, it is known that that many market indices exhibit volatility clustering.

Very interesting behavior was observed in the case where the information on the volatility feed back.

\textit{2. Volatility Feedback}

Small time scales in simulations of the urn model.

There is a very similar slow convergence to Gaussian behavior with superdiffusive fluctuations. The apparent Levy distributions with $H < 1/2$ and $H = 1/2$ are a transient phenomenon for short time scales only, but there is slow convergence to the Gaussian process. Remarkably we observe the apparent Levy distributions with $H < 1/2$. Indeed, as referred to in the Introduction, it has recently been argued based on real economic data, that in the long run price variations are random walk, the exponent is larger than $1/2$. Indeed, we refer back to the Introduction that the effective Hurst factor for the previous model with $H = 1/2$ is referred to a logitnormal results to refer to, we speculate that the slow convergence may be due to a logarithmic average, though the convergence is very slow. Although we presently have no analytical
The variations are more dramatic than in any of the other cases studied. As shown in Figure 5c, there is an apparent plateau with an exponent $H \approx 0$ even for relatively long time scales. However, when plotted in terms of properly defined scaling variables, the picture becomes immensely simplified. Figure 5e shows the same data plotted as a function of the scaling variable $z = \frac{dp}{(dt)^0.65}$. In the Figure, the vertical axis is scaled so that the entire distribution is normalized to 1. Within statistical uncertainty, all the curves collapse onto a single curve. This data collapse shows that the distribution of price variations exhibits scaling behavior.

$$P(dp, dt) \sim (dp, dt)^{0.65}$$

In order to study the scaling behavior further, we plot the distribution of price differences for time intervals $dt$ ranging from 200 to 6400. The time intervals over which the data spans the interval from $dt = 200$ to $dt = 6400$. The exponent in the scaling function is very similar to the one observed for real markets. The value of the exponent in the scaling function is precisely a Levy function with power law decay. The data is insufficient to determine whether the distribution shows a fat tail indicative of a significant probability of observing fluctuations Gaussian with the same variance as also shown. Note the dramatic difference. In particular, for comparison a function $F$ is a stable Levy distribution.

This was precisely the behavior observed by Mandelbrot, who suggested that the scaling

\[ P = f(z) \]

exhibits scaling behavior.
The noise traders are not aware of who is a noise trader and who is rational. When
they happen to buy, their potential selling price will be an amount $p_{\min}$ higher than
their buying price, which become relevant if they happen to sell their share is also fixed throughout
the simulation. They are an amount $p_{\min}$ lower than distributed randomly between $I$ and
their potential buying prices, which become relevant if they happen to sell their share is also fixed throughout
their potential buying. Mean $p$, limited by $p_{\min}$ and $p_{\max}$, Half of the rational traders own a share and their
in the initial setup the noise traders' prices are distributed in an interval around the
upper limit corresponds to a time interval during which each trader on average performs a

D. A Market with Fundamental Value Buyers

It is natural to consider what happens in a market with both rational optimizing agents
and Noise Traders with Imitating Behavior
At that point, all of the fundamental traders own stock and most of the noise traders are
subsequent the prices are initialized around 4000 on the horizontal axis of Figure 6
and common sense is rapidly restored. Figure 7 shows the skew distribution of prices
and traders is confined within the range of the rational traders. Only very brief develop-
Figure 6 shows the situation with 20% rational traders. Now, the price range of
law with exponent $H = 1/2$ just as for the case with noise traders only in the urn model.
are a market with few rational traders, the price variations at long time scales follow a power
a “bubble”. At least in our model, we observe that this is a possible outcome in
behavior, where the crowd effect leads to unreasonable high prices is
in Section II, the highest price can be thought of as the price with zero risk aversion, plus
and 2006. This interval is supposed to represent the interval spanned by the utility function
All the prices of the rational traders are arbitrarily confined to the interval between 1981

1. Numerical Simulation Results

In the beginning of the simulation, the prices of the two types of agents are similar, but
the stock rises.

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the stock rises.
De Long, Schleifer, Summers, and Waldmann (1990) in a stimulating paper, have considered an economy with optimizing agents and noise agents. In their analysis, they observed that under some circumstances the noise agents might outperform the rational optimizers. If the optimizers are risk averse, the presence of random agents adds to the variance in price. The optimizers, due to their risk aversion, may be more cautious than they would be otherwise.

In order to estimate the dividends relative to the interest rate, we assume that the highest selling price, $p_s$, among all the rational agents is the price for which the dividend, on average, balances the interest rate. Thus, if you buy at a lower price, you make a profit by keeping the dividend. If you buy at a higher price, you make a loss.

The average capital gain for the fundamental value traders was 1.87 units, compared with a loss of 0.5 units for the noise traders. Thus, their activity is only 2% of that of the noise traders. Actually, only 3 out of 100 rational traders did not sell at all during the simulation.

There are two sources of profits, dividends and capital gains. The average capital gain for the noise traders is -0.72 units, the average profit from dividends is 0.00000001 units, the average profit from trading is 0.00000001 units, the average profit from trading is 0.00000001 units. The difference $2120 - d$ in the bank. With an interest rate $d$ paid every $t$ time steps, the profit per time step. Assuming some shorthand notation that $t! = 1/t!$ per time step. Thus, if you buy a share for $d$ you make a profit by keeping the dividends. If you buy at a lower price, you make a profit by keeping the dividends. If you sell at a higher price, you make a loss.

In the simulation, extending over 1 million time units, each agent was updated 1 million times. The noise traders have traded on average 1400 times, while the fundamental value traders, who trade only 3 times, have traded on average 80 times.

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the rational traders were/8/3 units. Thus, there appears to be no significant difference. This is due to the fact that all the trading action is rather close to the equilibrium value of/2/0/0/0, so the average profit from dividends for all agents during the/1/0/0/0/0 time steps is/7/5, corresponding to half the agents holding shares and making profits at any given time. Thus, there appears to be no significant difference. Thus the rational traders was/35 units. This is due to the fact that the agents are too close to the equilibrium value of/2/0/0/0. However, their uncertainty about the behavior of the noise traders still remains. Keynes' observation about wanting to know what the average opinion of what the average opinion will be still holds. Dubey, Cahanopoulos, and Stinblik (1987) were able to show analytically how ever, this statement is deceiving. The profits from dividends of some rational agents were twice as large, obtained by simply holding on to the stock throughout the simulation.
pences may come through the use of intermediaries. One of the little understood phenomena.

Another potential source to consider is contributing to the dynamics of stock market

A. Experts, Intermediaries, and Social Networks

and phenomena which merit consideration.

of a highly complex set of problems. In this section we briefly comment on other approaches.

The models presented in this section provide an opportunity only to scratch the surface

V. MANY MODELS OF MARKET PRICING

investigation of trade with many stocks to future investigation.

there may be stock trading taking place, unlike the case for a single stock. We defer the

conversely claims to the same single consumer good. Furthermore, at an equilibrium point,

may longer be unattainable even with trade in only two different stocks, both of which pay out in

stocks. The noncooperative equilibrium of the strategic market game for the optimizers only

note that a distinctly new phenomenon appears in proceeding from one to two or more

natural generalization of our modelling would be to consider trade with many securities. We

For simplicity we have limited our considerations thus far to trading in one security.

C. Trading with Many Stock

while others sell lottery tickets or own slot machines.

are after all individuals who buy lottery tickets or play slot machines for their entire lives,

seen in the numerical simulations above. Even in games which are zero sum in money, there

in predator-prey species relationships. It is feasible for both species of traders to survive as

A simplistic view of competition would be that the optimizers and experts would even-

P. Survival of the Fittest?
of communication networks in coordinating human behavior.

Learning (Cage and Rosenbluth, 1966), and the work in contagion all suggest the importance

The network in question returns to scale of Wright (1960), the study of imitation and social

and the stock market.

alone was around $600 billion or around 2.7% of trade, and the feedback between the firms

of fluctuations in the economy. They are the role of credit intermediaries in 1994 margin debt

same as the manufacturing firm. Let out of these models are several other basic sources

approximation one could have a birth death process for a financial intermediary. A first crude

brokerage houses, where often success depended on one expert individual. As a first crude

banks and insurance companies appear, on the whole, to have been longer lived than stock

intermediaries. Historically, many of the intermediaries were partnerships or mutual firms,

the best portfolio of the 3,000 firms. The variation in expertise also primarily in the 1,000

time to select the best of the 1,000 intermediaries who spend their time trading

extreme model has all 14,000 agents as secondary sensors who spend their time trading.

A natural extension of our models is to include an extra layer of intermediaries. An

information and, in some instances, expertise.

supplies credit, liquidity, matching, accounting, record-keeping, transaction costs savings,

1994, p.33). More and more the individual investor invests through an intermediary who

finds and insurance companies hold well over 50% of all equities (see NYSE Fact Book) |

financial intermediaries trading on the NYSE. Institutions such as pension funds, mutual |

In the few statistics noted in Section III we observed that there are around 1,000 |

particles were all simultaneously polarized.

of crowds. When a panic breaks out it is as though a mass of individually independent |

written over a hundred years ago is still possibly the most penetrating writing on the nature |

in social psychology is the behavior of the crowd. The classical work of Le Bon (1892) |

The classical work of Le Bon (1898), written over a hundred years ago is still possibly the most penetrating writing on the nature of the crowd. In social psychology is the behavior of the crowd.
B. Process Models, Dynamics, and Heterogeneous Agents

A key theme in our approach is the necessity to have a fully defined process model in the study of stock market trading. Much of the misunderstanding about rational expectations in the study of economics has been observed that it is not uncommon for a previously successful model to predict outcomes that are different from actual market behavior. A consistent process model of trade is considered. Recently in the study of economics, there has been a growing realization of the importance of studying models with agents that have heterogeneous characteristics (see for example, Grandmont, 1993) and with agents that have heterogeneous expectations (for example, Chamley, 1990). The assumption of rational expectations or non-cooperative equilibrium, by imposing prematurely consistency conditions on expectations, removes the degrees of freedom present in the actual dynamics, which could result in many cooperative equilibria. By imposing prematurely consistency conditions on expectations, the degrees of freedom are reduced. The empirical question of how people form expectations is faced and replaced by a convenient mathematical simplification that has to be rationalized as a good enough approximation.

C. Learning, Habit, and Optimization

In the work of Arthur, Holland, LeBaron, Palmer, and Taylor (1996), the agents are investors, bankruptcy experts, short sellers, day traders, and bond traders all have different amounts of learning needed to survive may not be too high. In the markets the long term environment and the heterogeneity of learning, if one can find the appropriate niche, then the sudden change to perfect adaptation. There is a trade-off between the variability in the industry to the one because the lesson learned yesterday that led to success is not understood since Chamberlin, it has been observed that it is not uncommon for a previously successful strategy to have habit takes over from active search. In finance there are the ideas of the market niche and financial boutiques. In the study of innovation and new products, habit takes over from active search. Simon, years ago, coined the term "satisficing" to test in a soup of different conjectures and rules of thumb. With the ideas of the market niche and financial boutiques, in the study of innovation and new products, habit takes over from active search.
D. The Importance of Time and Size Scales

...
VI. CONCLUDING REMARKS

Recently there has been a growth of relatively sophisticated behavioral models in eco-

moderate tendencies to treat all uncertainty as a lottery ticket. This treatment is

product is rarely of concern to fundamental long-term investors.

up immediately by the bankruptcy experts and few others. The successful selection of a new
information. Information on a new court settlement concerning an insolvent firm is picked
the announcement of a president probably influences all and few know how to assess the
exceptional events which trigger market reactions must be treated differently. For example,
in the actual economy, different specialists may be regarded as different sensors and thus the
In the models we investigate here, all exceptional uncertainty is in essence the same. But

E. Quantitative and Qualitative Aspects of Uncertainty

In the models we investigate here, all exogenous uncertainty is in essence the same. But

laws of transformation. When this procedure of modeling economic process is adopted, the
crude which can then be regarded as well-defined different basic particles described by the
are required. In particular, this approach considerably the role of money and various forms of
the study of equilibrium, a full process model is required and that simple rules of consetation
been drawn. No attempt is made here to push analogies beyond the observation that even in
numbers where the analogies to stochastic mass particle behavior in physics and biology have

In the models we investigate here, all exogenous uncertainty is in essence the same. But

laws of transformation. When this procedure of modeling economic process is adopted, the

crude which can then be regarded as well-defined different basic particles described by the

cess that, in general, may not be heading to a unique stationary state. In this model, the behavioral assumptions were limited to dividend acquisition and to simplistic behavior by those authors. A further extension of the type of model presented here is to have expectations of future stock prices depend on information networking and learning, or on some historical material and possibly on the prediction of some small subset of experts. Genocination, but not too much, to say about next year’s market.

The explicit introduction of birth and death processes for the agents provides a strong forcing function or set of exogenous events constantly applied to the system. A sand pile analogy is appropriate, suppose we have a table of sand with a constant stream of sand falling on it. Eventually, if we have a table of sand with a constant stream of sand falling on it, the amount of sand falling on the edge of the table will equal that of the sand being poured onto the table. Similarly, the births and deaths of individuals and products may balance out on the market. The rational expectations approach would have us believe that if some agents can perceive arbitrages that others miss, the arbitrages will be removed quickly and the brighter agents will capture all profits. A more ecological view point is consistent with the existence of some experts who make a good living in their expert roles. It is quite possible that there are virtuoso players in finance as there are in music. But there is little evidence that a Mozart or a Bach can found a dynasty of progeny with equal talent. For any length of time, the classical economic models which reflect knowledge of everything and little evidence that there are virtuoso players in finance as there are in music. But there is little evidence that a Mozart or a Bach can found a dynasty of progeny with equal talent. A recent development is consistent with the existence of some experts who make a good living in their expert roles. But that does not tell us about the micro behavior of the individual units.
at their home on Cape Cod where this work was completed. The authors have enjoyed stimulating discussions on economics with W. Nagel. This work was supported by the U.S. Department of Energy Division of Materials Sciences under Contract No. DE-AC02-76.

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FIGURES

The market (Fig. 1) is now self-organized, rather than limited by the boundary conditions on a walk walk value of H = 1/2 rather than H = 1/4. Note that the width of the distribution of prices in dynamical sense and the resulting asymptotic exponential for price variations is equal to the random traders when stocking new asking prices and bidding prices. The price fluctuations are much more

FIG. 1. The urn model. Data shown as in Figure 2. N=200, D=0.05, the agents copy existing curve.

FIG. 2. As Figure 2, but with an additional drift D=0.05 towards the market price.

The logarithm substantially attenuates the small t correction apparent in the upper curve. The local derivative of the upper curve, denoting the effective exponent H of the range R, shows the lower curve is relatively low. (c) Hurst plot of the range R of price variations versus time t. The lower curve is relatively low. (d) The full plot of the range R of price variations versus time t. The lower curve is relatively low. (e) The full plot of the range R of price variations versus time t. The lower curve is relatively low. (f) The full plot of the range R of price variations versus time t. The lower curve is relatively low.

FIG. 3. As Figure 2, but with an additional drift D=0.05 towards the market price.

FIG. 4. A-particles (white) diffuse from the left and B-particles (black) diffuse from the right. The annihilated particles are then fed back at opposite ends of the tube.
none of the rational traders owning shares.

prices at the anomalous low price around $t=100000$. At that point, the distribution is skewed, with the region spanned by the fundamental value traders, except for short glitches. (b) Distribution of price variations for $20\%$ fundamental value traders. Market prices are confined to a typical configuration of prices in the market.

fundamental value estimates. (b) Typical configuration of prices in the market.

market price exceeds anything that can be justified by a "bubble" around $t=200000$ where the market price exceeds anything that can be justified by fundamental value traders ask and bid prices in the range between 1.931 and 2.068. (a) Note the "bubble" around $t=200000$ where the market price exceeds anything that can be justified by fundamental value traders ask and bid prices in the range between 1.931 and 2.068. (a) Note the

fundamental value traders ask and bid prices in the range between 1.931 and 2.068. (a) Note the

having variances exceeding several standard deviations.

$a = 1600$ compared with a Gaussian. Note the fat tail, indicating a significant probability of scaling variable. The scaling exponent is $H = 0.66$, (f) Price fluctuations for a single value of $H$. (e) Scaling plot of price fluctuations as defined in text. All curves can be described by a single plateau where the exponent $H = 0.66$. (d) Distribution of price fluctuations for various time intervals in the observed price variation over the previous 30 time steps. $N = 10000, D = 0.2$. Note the wide variance feedback in the urn model. (e) Diffusion of prices for each agent is equal.